Lecture Notes

Hou, Mo, Xue, and Zhang (2019, Review of Finance, "Which Factors?")

Lu Zhang[1]

[1]Ohio State and NBER

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Ohio State

Many recently proposed, seemingly different factor models are closely related to the q-factor model

In spanning regressions, the q-factor model largely subsumes the Fama-French 5- and 6-factor models

The Stambaugh-Yuan factors sensitive to their construction, once replicated via the traditional approach, are close to the q-factors, with correlations of 0.8 and 0.84

The Daniel-Hirshleifer-Sun factors also sensitive to their construction, once replicated via the traditional approach, are close to the q-factors, with correlations of 0.69

Valuation theory predicts a positive relation between the expected investment and the expected return

Introduction

The 2019 Spängler IQAM Best Paper Prize for the best investments paper

Outline

Outline

The Playing Field

8 competing factor models

The q-factor model, the q^5 model

The Fama-French 5-factor model, the 6-factor model, the alternative 6-factor model with RMWc

The Stambaugh-Yuan 4-factor model

The Barillas-Shanken 6-factor model, including MKT, SMB, $R_{\mathrm{I/A}}$, R_{Roe}, the Asness-Frazzini monthly formed HML, UMD

The Daniel-Hirshleifer-Sun 3-factor model

$$E[R_i - R_f] = \beta^i_{\mathrm{MKT}}\, E[\mathrm{MKT}] + \beta^i_{\mathrm{Me}}\, E[R_{\mathrm{Me}}] + \beta^i_{\mathrm{I/A}}\, E[R_{\mathrm{I/A}}] + \beta^i_{\mathrm{Roe}}\, E[R_{\mathrm{Roe}}]$$

- MKT, R_{Me}, $R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and profitability (return on equity, Roe) factors, respectively
- β^i_{MKT}, β^i_{Me}, $\beta^i_{\mathrm{I/A}}$, and β^i_{Roe} are factor loadings

R_{ME}, $R_{\mathrm{I/A}}$, and R_{Roe} from independent, triple $2 \times 3 \times 3$ sorts on size, investment-to-assets, and Roe

Variable definitions:

- Size: Stock price times shares outstanding from CRSP
- Investment-to-assets, I/A: Annual changes in total assets (item AT) divided by lagged total assets
- Roe: Income before extraordinary items (item IBQ) divided by one-quarter-lagged book equity

NYSE breakpoints: 50-50 for size, 30-40-30 for I/A, and 30-40-30 for Roe; value-weighted returns

Timing:

- Annual sort in June on the market equity at the June end
- Annual sort in June of year t on I/A for the fiscal year ending in calendar year $t-1$
- Monthly sort at the beginning of each month on Roe with the most recently announced quarterly earnings

Results robust to all monthly sorts on size, I/A, and Roe

The Playing Field

Extending the q-factors backward to January 1967

Hou, Xue, and Zhang (2015) start in January 1972, restricted by earnings announcement dates and quarterly book equity data

Prior to January 1972, use the most recent earnings from the fiscal quarter ending at least 4 months prior to the portfolio formation

Maximize the coverage of quarterly book equity

The Playing Field

Backward extending the q-factors, maximize the coverage of quarterly book equity

Use quarterly book equity whenever available

Supplement the coverage for fiscal quarter 4 with book equity from Compustat annual files

If available, backward impute beginning-of-quarter book equity as end-of-quarter book equity minus quarterly earnings plus quarterly dividends

Finally, forward impute $\mathrm{BEQ}_t = \mathrm{BEQ}_{t-j} + \mathrm{IBQ}_{t-j+1,t} - \mathrm{DVQ}_{t-j+1,t}$, in which BEQ_{t-j} is the latest available quarterly book equity as of quarter t, $\mathrm{IBQ}_{t-j+1,t}$ and $\mathrm{DVQ}_{t-j+1,t}$ the sum of quarterly earnings and the sum of quarterly dividends from quarter $t-j+1$ to quarter t, respectively, and $1 \leq j \leq 4$

Augment the q-factor model with the expected growth factor to form the q^5 model:

$$
\begin{aligned}
E[R_i - R_f] \quad = \quad & \beta^i_{\mathrm{MKT}}\, E[\mathrm{MKT}] + \beta^i_{\mathrm{Me}}\, E[R_{\mathrm{Me}}] \\
& + \beta^i_{\mathrm{I/A}}\, E[R_{\mathrm{I/A}}] + \beta^i_{\mathrm{Roe}}\, E[R_{\mathrm{Roe}}] + \beta^i_{\mathrm{Eg}}\, E[R_{\mathrm{Eg}}]
\end{aligned}
$$

Stress-tests from a large set of 150 anomalies show that the q^5 model improves on the q-factor model substantially

Forecast $\mathrm{d}^{\tau}\mathrm{I/A}$, τ-year ahead investment-to-assets changes, via monthly cross-sectional regressions

Motivating predictors based on a priori conceptual arguments (internal funds available for investments, accounting conservatism, short-term dynamics of investment growth):

- Tobin's q
- Cash flows
- Change in return on equity

R_{Eg} from monthly, independent 2×3 sorts on size and $E_t[\mathrm{d}^1\mathrm{I/A}]$

The Fama-French 5-factor model:

$$E[R_i - R_f] = b_i\, E[\text{MKT}] + s_i\, E[\text{SMB}] + h_i\, E[\text{HML}] \\ + r_i\, E[\text{RMW}] + c_i\, E[\text{CMA}]$$

- MKT, SMB, HML, RMW, and CMA are the market, size, value, profitability, and investment factors, respectively
- b_i, s_i, h_i, r_i, and c_i are factor loadings

Fama and French (2018) add UMD to form the 6-factor model

The Playing Field

Timeline: The q-factor model predates the Fama-French 5-factor model

Neoclassical factors	July 2007
An equilibrium three-factor model	January 2009
Production-based factors	April 2009
A better three-factor model that explains more anomalies	June 2009
An alternative three-factor model	April 2010, April 2011
Digesting anomalies: An investment approach	October 2012, August 2014
Fama and French (2013): A four-factor model for the size, value, and profitability patterns in stock returns	June 2013
Fama and French (2014): A five-factor asset pricing model	November 2013, September 2014

Start with two clusters of anomalies:

- MGMT: net stock issues, composite issues, accruals, net operating assets, asset growth, and change in gross PPE and inventory scaled by lagged book assets
- PERF: failure probability, O-score, momentum, gross profitability, and return on assets

Form composite scores by equal-weighting a stock's percentiles in each cluster (realigned to yield average L$-$H returns > 0)

Form the MGMT and PERF factors from independent 2×3 sorts by interacting size with each composite score

The Playing Field

Stambaugh and Yuan deviate from the traditional construction in important ways

The NYSE-Amex-NASDAQ 20–80 breakpoints, as opposed to the NYSE 30–70 breakpoints

The size factor contains stocks only in the middle portfolios of the double sorts, as opposed to from all portfolios

Use their original factors, as well as replicated factors via the traditional construction

Results are sensitive to the construction method

FIN based on 1-year net share issuance and 5-year composite issuance; PEAD on 4-day cumulative abnormal return around the most recent quarterly earnings announcement, Abr

Factor construction also deviates from the more common approach:

- NYSE 20–80, as opposed to NYSE 30–70, breakpoints
- Abr only, as opposed to Abr, Sue, and Re per Chan, Jegadeesh, and Lakonishok (1996)
- More ad hoc, involved sorts on FIN

Use reproduced and replicated factors (NYSE 30–70 breakpoints on the composite scores of FIN from combining net share and composite issuances and of PEAD from combining Abr, Sue, and Re by equal-weighting a stock's percentile rankings)

Outline

Rely mostly on spanning tests as an informative and concise way to compare factor models on empirical grounds

Barillas and Shanken (2017, 2018): For traded factors, the extent to which each model is able to price the factors in the other model is all that matters for comparison; testing assets irrelevant

In complementary work, Hou, Mo, Xue, and Zhang (2021) stress-test factor models with a large set of 150 significant anomalies, with results consistent with our spanning tests

Spanning Tests

The Fama-French 5- and 6-factor models cannot explain the q and q^5 factors, 1/1967–12/2016

	$\overline{R}$	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{UMD}	β_{RMWc}
R_{Me}	0.31	0.05	0.01	0.97	0.04	−0.03	0.02		
	2.43	1.53	0.88	68.35	1.85	−0.91	0.66		
		0.03	0.01	0.97	0.06	−0.03	0.01	0.03	
		0.85	1.35	71.18	3.01	−1.28	0.29	2.54	
		0.05	0.01	0.96	0.05		0.01	0.03	−0.07
		1.37	0.62	74.88	2.92		0.51	2.75	−2.28
$R_{I/A}$	0.41	0.12	0.01	−0.05	0.04	0.07	0.80		
	4.92	3.44	0.91	−3.19	1.63	2.48	29.30		
		0.11	0.01	−0.05	0.05	0.06	0.80	0.01	
		3.11	1.09	−3.17	2.12	2.22	30.79	0.82	
		0.11	0.01	−0.05	0.05		0.78	0.01	0.06
		2.78	1.10	−3.13	2.22		27.89	0.81	1.49

Spanning Tests

The Fama-French 5- and 6-factor models cannot explain the q and q^5 factors, 1/1967–12/2016

	$\overline{R}$	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{UMD}	β_{RMWc}
R_{Roe}	0.55	0.47	−0.03	−0.12	−0.24	0.70	0.10		
	5.25	5.94	−1.20	−2.92	−3.75	12.76	1.01		
		0.30	−0.00	−0.12	−0.10	0.65	−0.02	0.24	
		4.51	−0.01	−3.66	−2.04	14.69	−0.24	9.94	
		0.23	0.03	−0.10	−0.03		−0.18	0.24	0.72
		2.80	1.41	−2.49	−0.49		−2.05	7.12	8.49
R_{Eg}	0.82	0.78	−0.10	−0.14	−0.08	0.25	0.28		
	9.81	11.34	−5.62	−5.36	−2.62	5.19	5.43		
		0.70	−0.09	−0.14	−0.02	0.22	0.22	0.12	
		11.10	−5.43	−6.43	−0.54	5.43	5.12	6.42	
		0.61	−0.06	−0.10	−0.00		0.18	0.11	0.39
		9.33	−3.41	−4.01	−0.01		3.87	5.77	6.73

Spanning Tests

The Fama-French 5- and 6-factor models cannot explain the q and q^5 factors, the Gibbons-Ross-Shanken (GRS) test, 1/1967–12/2016

	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$			$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$		
	FF5	FF6	FF6c	FF5	FF6	FF6c
GRS	22.72	14.60	8.20	55.14	48.85	36.59
p	0.00	0.00	0.00	0.00	0.00	0.00

Spanning Tests

The q and q^5 models largely subsume the Fama-French 5- and 6-factor models, 1/1967–12/2016

	$\overline{R}$	α	β_{MKT}	β_{Me}	$\beta_{\mathrm{I/A}}$	β_{Roe}	β_{Eg}
SMB	0.25	0.04	−0.01	0.95	−0.08	−0.09	
	1.93	1.42	−0.82	60.67	−4.48	−6.00	
		0.07	−0.01	0.94	−0.07	−0.08	−0.04
		2.29	−1.32	61.42	−3.86	−4.44	−1.95
HML	0.37	0.07	−0.04	0.02	1.02	−0.19	
	2.71	0.62	−0.96	0.24	12.11	−2.61	
		0.05	−0.03	0.02	1.01	−0.20	0.03
		0.48	−0.90	0.26	11.50	−2.42	0.36
UMD	0.65	0.12	−0.08	0.23	−0.00	0.91	
	3.61	0.50	−1.25	1.73	−0.02	5.90	
		−0.16	−0.03	0.27	−0.11	0.78	0.44
		−0.78	−0.51	2.00	−0.60	4.40	2.62

Spanning Tests

The q and q^5 models largely subsume the Fama-French 5- and 6-factor models, 1/1967–12/2016

	$\overline{R}$	α	β_{MKT}	β_{Me}	$\beta_{\mathrm{I/A}}$	β_{Roe}	β_{Eg}
CMA	0.33	−0.00	−0.05	0.04	0.96	−0.10	
	3.51	−0.02	−3.77	1.91	33.56	−3.57	
		−0.04	−0.04	0.05	0.94	−0.12	0.06
		−0.96	−3.14	2.12	35.60	−3.89	2.07
RMW	0.26	0.01	−0.03	−0.12	0.03	0.54	
	2.50	0.08	−1.17	−1.71	0.38	8.50	
		−0.01	−0.03	−0.12	0.02	0.53	0.03
		−0.16	−0.13	−1.59	0.28	7.85	0.42
RMWc	0.33	0.25	−0.10	−0.18	0.09	0.29	
	4.16	3.83	−6.00	−5.25	2.02	9.88	
		0.14	−0.09	−0.17	0.05	0.23	0.18
		2.18	−5.15	−4.45	0.93	6.55	4.27

Spanning Tests

The q and q^5 models largely subsume the Fama-French 5- and 6-factor models, the GRS test, 1/1967–12/2016

	$\alpha_{\text{HML}}, \alpha_{\text{CMA}}, \alpha_{\text{RMW}} = 0$		$\alpha_{\text{RMW}}, \alpha_{\text{UMD}}$ $\alpha_{\text{RMW}}, \alpha_{\text{UMD}} = 0$		$\alpha_{\text{HML}}, \alpha_{\text{CMA}}, \alpha_{\text{RMWc}}, \alpha_{\text{UMD}} = 0$	
	q	q^5	q	q^5	q	q^5
GRS	0.20	0.62	0.36	0.65	6.14	1.81
p	0.90	0.60	0.84	0.62	0.00	0.13

Spanning Tests

Explaining the q and q^5 factors with the original Stambaugh-Yuan model, 1/1967–12/2016

	$\overline{R}$	α	MKT	SMB	MGMT	PERF
R_{Me}	0.31	−0.04	−0.01	0.97	−0.06	−0.06
	2.43	−0.65	−0.67	25.97	−1.71	−2.98
$R_{\text{I/A}}$	0.41	0.08	0.01	0.05	0.53	−0.02
	4.92	1.26	0.52	2.35	15.99	−1.06
R_{Roe}	0.55	0.33	0.02	−0.20	0.02	0.42
	5.25	3.55	0.73	−3.44	0.42	11.65
R_{Eg}	0.82	0.55	−0.03	−0.10	0.29	0.21
	9.81	9.04	−1.76	−3.92	12.19	10.72

	$\alpha_{\text{I/A}}, \alpha_{\text{Roe}} = 0$	$\alpha_{\text{I/A}}, \alpha_{\text{Roe}}, \alpha_{\text{Eg}} = 0$
GRS	8.16	30.24
p	0.00	0.00

Spanning Tests

Explaining the q and q^5 factors with the replicated Stambaugh-Yuan model, 1/1967–12/2016

	$\overline{R}$	α	MKT	SMB	MGMT	PERF
R_{Me}	0.31	0.01	−0.04	0.95	−0.03	0.10
	2.43	0.18	−2.51	29.43	−1.00	4.23
$R_{\mathrm{I/A}}$	0.41	0.07	0.00	0.05	0.70	−0.02
	4.92	1.41	−0.08	2.77	26.78	−0.85
R_{Roe}	0.55	0.32	0.01	−0.16	−0.04	0.59
	5.25	4.71	0.50	−4.54	−0.82	20.03
R_{Eg}	0.82	0.58	−0.05	−0.09	0.35	0.25
	9.81	10.25	−3.29	−4.48	13.57	9.03

	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$
GRS	12.12	41.27
p	0.00	0.00

Spanning Tests

Explaining the original Stambaugh-Yuan factors with the q and q^5 models, 1/1967–12/2016

	$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}
SMB	0.44	0.16	0.01	0.86	−0.01	0.01	
	3.60	3.37	0.57	31.16	−0.23	0.45	
		0.14	0.01	0.87	−0.02	−0.00	0.04
		2.43	0.81	30.92	−0.50	−0.03	0.97
MGMT	0.61	0.36	−0.17	−0.15	1.00	−0.06	
	4.72	4.73	−7.95	−5.02	18.59	−1.33	
		0.12	−0.13	−0.11	0.90	−0.18	0.38
		1.64	−6.70	−4.15	18.76	−3.91	7.61
PERF	0.68	0.34	−0.18	0.11	−0.30	0.95	
	4.20	2.00	−4.22	1.35	−2.02	10.42	
		0.01	−0.12	0.15	−0.44	0.79	0.53
		0.05	−3.17	1.95	−3.06	8.40	4.80

	$\alpha_{\mathrm{MGMT}}, \alpha_{\mathrm{PERF}} = 0$ in q	$\alpha_{\mathrm{MGMT}}, \alpha_{\mathrm{PERF}} = 0$ in q^5
GRS	17.16	1.46
p	0.00	0.23

Spanning Tests

Explaining the replicated Stambaugh-Yuan factors with the q and q^5 models, 1/1967–12/2016

	$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\text{I/A}}$	R_{Roe}	R_{Eg}
SMB	0.31	0.06	0.06	0.94	0.04	−0.16	
	2.13	1.13	3.37	18.96	0.86	−4.94	
		0.09	0.06	0.93	0.05	−0.15	−0.05
		1.72	3.28	18.52	1.08	−3.94	−1.54
MGMT	0.47	0.20	−0.09	−0.10	0.92	−0.06	
	4.68	3.59	−5.82	−4.10	22.65	−1.68	
		−0.02	−0.05	−0.07	0.83	−0.17	0.36
		−0.38	−4.21	−3.30	23.50	−5.28	9.79
PERF	0.49	0.03	−0.08	0.08	−0.15	1.00	
	3.67	0.28	−2.87	1.85	−1.72	13.97	
		−0.19	−0.05	0.11	−0.24	0.89	0.35
		−1.87	−1.62	2.63	−2.91	11.57	4.85

	$\alpha_{\text{MGMT}}, \alpha_{\text{PERF}} = 0$ in q	$\alpha_{\text{MGMT}}, \alpha_{\text{PERF}} = 0$ in q^5
GRS	7.96	2.38
p	0.00	0.09

Spanning Tests

Explaining the q and q^5 factors with the reproduced Daniel-Hirshleifer-Sun model, 7/1972–12/2016

	$\overline{R}$	α	MKT	FIN	PEAD
R_{Me}	0.27	0.46	0.06	−0.24	−0.04
	2.03	3.11	1.10	−2.23	−0.28
$R_{\mathrm{I/A}}$	0.41	0.18	−0.03	0.29	−0.01
	4.69	2.56	−1.33	10.21	−0.21
R_{Roe}	0.54	0.10	0.01	0.24	0.38
	4.80	0.83	0.17	4.15	3.66
R_{Eg}	0.83	0.56	−0.08	0.22	0.21
	9.44	7.42	−4.49	8.36	5.20

	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$
GRS	4.89	23.90
p	0.01	0.00

Spanning Tests

Explaining the q and q^5 factors with the replicated Daniel-Hirshleifer-Sun model, 1/1967–12/2016

	$\overline{R}$	α	MKT	FIN	PEAD
R_{Me}	0.31	0.63	0.00	−0.46	−0.24
	2.43	4.25	0.07	−3.76	−3.20
$R_{\mathrm{I/A}}$	0.41	0.32	0.00	0.44	−0.07
	4.92	4.34	−0.14	8.97	−1.99
R_{Roe}	0.55	−0.14	0.04	0.32	0.78
	5.25	−1.91	1.65	5.98	18.90
R_{Eg}	0.82	0.54	−0.08	0.28	0.31
	9.81	7.45	−4.64	8.26	8.59

	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$
GRS	14.27	35.37
p	0.00	0.00

Spanning Tests

Explaining the reproduced Daniel-Hirshleifer-Sun factors with the q and q^5 models, 7/1972–12/2016

	$\overline{R}$	α	MKT	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}
FIN	0.83	0.33	−0.17	−0.21	1.15	0.33	
	4.55	2.67	−4.11	−2.36	11.45	3.89	
		0.14	−0.14	−0.19	1.08	0.24	0.30
		1.12	−3.47	−2.02	10.77	2.57	3.50
PEAD	0.62	0.56	−0.04	0.05	−0.08	0.19	
	7.73	5.66	−1.64	0.84	−1.06	3.53	
		0.47	−0.03	0.06	−0.11	0.15	0.15
		5.32	−1.17	1.02	−1.42	2.15	1.95

	$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q	$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q^5
GRS	29.67	14.99
p	0.00	0.00

Spanning Tests

Explaining the replicated Daniel-Hirshleifer-Sun factors with the q and q^5 models, 1/1967–12/2016

	$\overline{R}$	α	MKT	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}
FIN	0.32	0.00	−0.16	−0.22	0.86	0.22	
	2.53	0.01	−6.90	−3.94	14.01	4.23	
		−0.05	−0.15	−0.22	0.84	0.19	0.09
		−0.65	−6.97	−3.61	12.37	3.26	1.45
PEAD	0.72	0.43	0.00	0.02	−0.11	0.61	
	7.78	5.13	0.00	0.52	−1.71	11.76	
		0.31	0.02	0.03	−0.15	0.55	0.18
		4.07	0.96	0.98	−2.36	8.98	2.89

	$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q	$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q^5
GRS	20.44	8.67
p	0.00	0.00

Spanning Tests

The q and q^5 models versus the Barillas-Shanken 6-factor model, 1/1967–12/2016

Explaining the q^5 factors on the Barillas-Shanken factors

	$\overline{R}$	α	MKT	SMB	$R_{\mathrm{I/A}}$	R_{Roe}	UMD	$\mathrm{HML^m}$
R_{Me}	0.31	−0.04	0.02	1.00	0.03	0.09	0.02	0.05
	2.43	−1.08	1.79	60.21	1.11	2.98	1.85	2.01
R_{Eg}	0.82	0.60	−0.10	−0.11	0.18	0.25	0.09	0.06
	9.81	8.78	−5.80	−4.77	4.50	5.90	3.54	2.00

Explaining the Asness-Frazzini HML factor on the q models

	$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}
$\mathrm{HML^m}$	0.34	0.37	−0.01	−0.10	0.93	−0.69	
	2.13	2.36	−0.12	−0.95	8.18	−6.78	
		0.41	−0.01	−0.10	0.95	−0.67	−0.08
		2.99	−0.30	−0.98	7.72	−5.61	−0.72

The monthly formed q and q^5 models yield alphas of 0.18 ($t = 0.97$) and 0.26 ($t = 1.64$), respectively

Spanning Tests

Correlation matrix, 1/1967–12/2016

	SMB	HML	RMW	CMA	UMD	RMWc	MGMT	PERF	FIN	PEAD	HML^m
R_{Mkt}	0.28	−0.27	−0.24	−0.40	−0.15	−0.48	−0.49	−0.23	−0.57	−0.11	−0.12
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
R_{Me}	0.97	−0.04	−0.37	−0.05	−0.02	−0.53	−0.28	−0.20	−0.44	−0.18	0.00
	0.00	0.30	0.00	0.21	0.59	0.00	0.00	0.00	0.00	0.00	0.94
$R_{I/A}$	−0.19	0.67	0.10	0.91	0.03	0.26	0.84	−0.02	0.69	−0.07	0.49
	0.00	0.00	0.02	0.00	0.53	0.00	0.00	0.55	0.00	0.10	0.00
R_{Roe}	−0.37	−0.14	0.67	−0.09	0.50	0.57	0.05	0.80	0.34	0.69	−0.45
	0.00	0.00	0.00	0.03	0.00	0.00	0.25	0.00	0.00	0.00	0.00
R_{Eg}	−0.42	0.19	0.43	0.33	0.35	0.59	0.54	0.51	0.54	0.40	−0.06
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18

Outline

The q and q^5 models motivated from the investment CAPM (Zhang 2017)

The Fama-French 6-factor, Stambaugh-Yuan, Daniel-Hirshleifer-Sun, and Barillas-Shanken models all statistical

"We include momentum factors (somewhat reluctantly) now to satisfy insistent popular demand. We worry, however, that opening the game to factors that seem empirically robust but lack theoretical motivation has a destructive downside: the end of discipline that produces parsimonious models and the beginning of a dark age of data dredging that produces a long list of factors with little hope of sifting through them in a statistically reliable way (Fama and French 2018, p. 237)."

Valuation Theory

Can the 5-factor model be motivated from valuation theory?

Fama and French (2015) attempt to motivate their 5-factor model from the Miller-Modigliani (1961) valuation model:

$$\frac{P_{it}}{B_{it}} = \frac{\sum_{\tau=1}^{\infty} E[Y_{it+\tau} - \triangle B_{it+\tau}]/(1+r_i)^{\tau}}{B_{it}},$$

Fama and French derive three predictions, all else equal:

- A lower P_{it}/B_{it} means a higher r_i
- A higher $E[Y_{it+\tau}]$ means a higher r_i
- A higher $E[\triangle B_{it+\tau}]/B_{it}$ means a lower r_i

Fama and French (2015, p. 2): “Most asset pricing research focuses on short-horizon returns—we use a one-month horizon in our tests. If each stock’s short-horizon expected return is positively related to its internal rate of return—if, for example, the expected return is the same for all horizons—the valuation equation...”

Assumption clearly contradicting price and earnings momentum

Evidence on IRRs $\neq$ the one-period-ahead expected return, Hou, van Dijk, and Zhang (2012), Tang, Wu, and Zhang (2014)

Valuation Theory

I: IRR estimates for the Fama-French 5-factors, 1967–2016

	IBES earnings forecasts			Cross-sectional earnings forecasts			Cross-sectional Roe forecasts		
	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff
SMB	1.44	1.72	−0.28	2.53	3.22	−0.69	2.90	−0.25	3.15
	0.76	10.74	−0.15	1.23	5.60	−0.35	1.49	−0.93	1.64
HML	2.90	2.04	0.86	3.52	5.31	−1.79	3.60	5.14	−1.54
	1.28	9.07	0.39	1.88	25.28	−0.97	1.96	17.72	−0.85
RMW	4.52	−1.58	6.10	3.61	−1.84	5.45	3.14	−2.47	5.61
	2.88	−9.66	3.90	2.66	−9.41	4.07	2.54	−21.47	4.52
CMA	3.40	1.16	2.24	3.81	2.64	1.17	3.44	2.02	1.43
	2.92	7.09	2.02	3.34	19.06	1.04	3.17	13.47	1.34

Valuation Theory

II: HML is redundant in describing average returns in the data

HML redundant once CMA is included in the data per Fama and French (2015), inconsistent with their reasoning

Consistent with the investment CAPM:

$$E_t[r^S_{it+1}] = \frac{E_t[X_{it+1}]}{1 + a(I_{it}/A_{it})},$$

in which the denominator $= P_{it}/B_{it}$

Consistent with valuation theory too: Investment forecasts returns via P_{it}/B_{it}, not $E_t\left[\triangle B_{it+\tau}/B_{it}\right]$ as advertised by Fama and French

Valuation Theory

III: The expected investment-return relation is likely positive

Reformulating valuation theory with $E_t[r_{it+1}]$:

$$
\begin{aligned}
P_{it} &= \frac{E_t[Y_{it+1} - \triangle Be_{it+1}] + E_t[P_{it+1}]}{1 + E_t[r_{it+1}]}, \\
\frac{P_{it}}{Be_{it}} &= \frac{E_t\left[\frac{Y_{it+1}}{Be_{it}}\right] - E_t\left[\frac{\triangle Be_{it+1}}{Be_{it}}\right] + E_t\left[\frac{P_{it+1}}{Be_{it+1}}\left(1 + \frac{\triangle Be_{it+1}}{Be_{it}}\right)\right]}{1 + E_t[r_{it+1}]}, \\
\frac{P_{it}}{Be_{it}} &= \frac{E_t\left[\frac{Y_{it+1}}{Be_{it}}\right] + E_t\left[\frac{\triangle Be_{it+1}}{Be_{it}}\left(\frac{P_{it+1}}{Be_{it+1}} - 1\right)\right] + E_t\left[\frac{P_{it+1}}{Be_{it+1}}\right]}{1 + E_t[r_{it+1}]}.
\end{aligned}
$$

Recursive substitution: A positive $E_t[\triangle B_{it+\tau}/B_{it}]$-$E_t[r_{it+1}]$ relation, consistent with the investment CAPM

After arguing for a negative $E_t\left[\triangle Be_{it+\tau}/Be_{it}\right]$-$E_t\left[r_{it+1}\right]$ relation, Fama and French (2015) use current asset growth $\triangle A_{it}/A_{it-1}$ to proxy for $E\left[\triangle Be_{it+\tau}\right]/Be_{it}$

However, past assets (book equity) growth does not forecast future book equity growth (while profitability forecasts future profitability)

See the lumpy investment literature, e.g., Dixit and Pindyck (1994); Domes and Dunne (1998); Whited (1998)

Valuation Theory

IV: Past investment is a poor proxy for the expected investment, 1963–2016

Total assets ≥ \$5mil and book equity ≥ \$2.5mil

	$\frac{Be_{it+\tau}-Be_{it+\tau-1}}{Be_{it+\tau-1}}$	$\frac{\triangle A_{it}}{A_{it-1}}$		$\frac{Be_{it+\tau}-Be_{it+\tau-1}}{Be_{it+\tau-1}}$	$\frac{\triangle Be_{it}}{Be_{it-1}}$		$\frac{Op_{it+\tau}}{Be_{it+\tau}}$	$\frac{Op_{it}}{B_{it}}$	
τ	γ_0	γ_1	R^2	γ_0	γ_1	R^2	γ_0	γ_1	R^2
1	0.09	0.22	0.05	0.09	0.20	0.06	0.03	0.80	0.54
2	0.10	0.10	0.01	0.10	0.10	0.02	0.05	0.67	0.36
3	0.10	0.06	0.01	0.10	0.06	0.01	0.07	0.59	0.27
4	0.10	0.05	0.00	0.10	0.05	0.00	0.09	0.53	0.22
5	0.10	0.04	0.00	0.10	0.02	0.00	0.10	0.49	0.19
6	0.10	0.05	0.00	0.10	0.03	0.00	0.11	0.45	0.16
7	0.09	0.04	0.00	0.10	0.03	0.00	0.11	0.43	0.15
8	0.09	0.03	0.00	0.10	0.01	0.00	0.12	0.40	0.13
9	0.09	0.03	0.00	0.10	0.01	0.00	0.12	0.39	0.12
10	0.09	0.04	0.00	0.09	0.02	0.00	0.12	0.38	0.11

Summary

Hou, Mo, Xue, and Zhang (2019, "Which factors?")

The q-factor model has emerged as a new workhorse model