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An Augmented *q*-Factor Model with Expected Growth*

Kewei Hou[1,2], Haitao Mo[3], Chen Xue[4], and Lu Zhang[1,5]

[1]The Ohio State University, [2]China Academy of Financial Research, [3]Louisiana State University, [4]University of Cincinnati and [5]National Bureau of Economic Research

Abstract

In the investment theory, firms with high expected investment growth earn higher expected returns than firms with low expected investment growth, holding investment and expected profitability constant. Building on cross-sectional growth

forecasts with Tobin's *q*, operating cash flows, and change in return on equity as predictors, an expected growth factor earns an average premium of 0.84% per month ($t = 10.27$) in the 1967–2018 sample. The q^5 model, which augments the Hou–Xue–Zhang (2015, *Rev. Finan. Stud.*, 28, 650–705) *q*-factor model with the expected growth factor, shows strong explanatory power in the cross-section and outperforms the Fama–French (2018, *J. Finan. Econom.*, 128, 234–252) six-factor model.

1. Introduction

In the investment theory, firms with high expected investment growth should earn higher expected returns than firms with low expected investment growth, holding current investment and expected profitability constant. Intuitively, if expected investment is high next period, the present value of cash flows from next period onward must be high. Consisting primarily of the present value of cash flows from next period onward, the benefit of investment this period must also be high. As such, if expected investment is high next period relative to current investment, the discount rate must be high to offset the high benefit of investment this period to keep current investment low.

To test this prediction, we perform cross-sectional forecasting regressions of investment-to-assets changes on current Tobin's *q*, operating cash flows, and the change in return on equity. Empirically, high cash flows and high changes in return on equity strongly predict high investment-to-assets changes, and high Tobin's *q* weakly predicts low investment-to-assets changes. The expected 1-year-ahead investment-to-assets changes closely track the average future realized 1-year-ahead investment-to-assets changes at the portfolio level.

More important, an independent 2×3 sort on size and expected 1-year-ahead investment-to-assets changes yields an expected investment growth factor, with an average premium of 0.84% per month ($t = 10.27$) from January 1967 to December 2018. The *q*-factor model cannot explain this factor premium, with an alpha of 0.67% ($t = 9.75$). As such, the expected growth factor represents a new dimension of the expected return variation that is largely missing from the *q*-factor model.

We augment the *q*-factor model with the expected growth factor to form the q^5 model and stress-test it along with other recent factor models. For testing deciles, we use a large set of 150 significant anomalies with New York Stock Exchange (NYSE) breakpoints and value-weighted returns from Hou, Xue, and Zhang (2019). For competing factor models, we examine the *q*-factor model; the Fama–French (2015) five-factor model; the Stambaugh–Yuan (2017) four-factor model; the Fama–French (2018) six-factor model; the Fama–French alternative six-factor model with the operating profitability factor, robust-minus-weak (RMW), replaced by a cash-based profitability factor, RMWc; the Barillas–Shanken (2018) six-factor model; as well as the Daniel–Hirshleifer–Sun (2019) three-factor model. The Barillas–Shanken specification includes the market factor, a size factor, the investment and return on equity (Roe) factors from the *q*-factor model, the Asness–Frazzini (2013) monthly formed high-minus-low (HML) factor, and the momentum factor (up-minus-down, UMD).

Improving on the *q*-factor model substantially, the q^5 model is the best performing model among all the factor models. Across the 150 anomalies, the average magnitude of the high-minus-low alphas is 0.19% per month, dropping from 0.28% in the *q*-factor model. The number of significant high-minus-low alphas ($|t| \geq 1.96$) is 23 in the q^5 model (6 with $|t| \geq 3$), dropping from 52 in the *q*-factor model (25 with $|t| \geq 3$). The number of rejections by the Gibbons, Ross, and Shanken (1989, hereafter GRS) test is also smaller, 57 versus 101. The q^5 model improves on the *q*-factor model across most anomaly categories, especially in the investment and profitability categories.

The *q*-factor model already compares well with the Fama–French six-factor model. The average magnitude of the high-minus-low alphas is 0.3% per month in the six-factor model (0.28% in the *q*-factor model). The numbers of significant high-minus-low six-factor alphas are 74 with $|t| \geq 1.96$ and 37 with $|t| \geq 3$. Both are higher than 52 and 25 in the *q*-factor model, respectively. However, the number of rejections by the GRS test is 91, which is lower than 101 in the *q*-factor model. Replacing RMW with RMWc improves the six-factor model's performance. The average magnitude of the high-minus-low alphas falls to 0.27%. The number of significant high-minus-low alphas drops to 59 with $|t| \geq 1.96$ but is still higher than 52 in the *q*-factor model. The number of rejections by the GRS test is 71. Although substantially lower than 101 in the *q*-factor model, the number of rejections is higher than 57 in the q^5 model.

The Stambaugh–Yuan model is comparable with the *q*-factor model. The number of high-minus-low alphas with $|t| \geq 1.96$ is 64, which is higher than 52 in the *q*-factor model. However, the number of rejections by the GRS test is 87, which is lower than 101 in the *q*-factor model. The Barillas–Shanken six-factor model performs poorly. The numbers of significant high-minus-low alphas are 63 with $|t| \geq 1.96$ and 37 with $|t| \geq 3$, and the number of rejections by the GRS test is 132 (out of 150 anomalies). Exacerbating the value-versus-growth anomalies, the Daniel–Hirshleifer–Sun three-factor model also performs poorly, with the second highest average magnitude of high-minus-low alphas, 0.37% per month, and the highest mean absolute alpha, 0.14%.

Our work makes two major contributions. First, we bring expected growth to the fore of empirical finance. This extension resolves many empirical difficulties of the *q*-factor model, such as the anomalies based on R&D-to-market as well as operating and discretionary accruals. Intuitively, R&D expenses depress current earnings but induce future growth. Also, given the level of earnings, high accruals imply low cash flows (internal funds available for investments) and, consequently, low expected growth.

Second, we conduct a large horse race of factor models. In contrast to small sets of testing portfolios in prior studies, we increase the number of testing anomalies drastically to 150. Barillas and Shanken (2018) conduct Bayesian tests with only eleven factors and downplay the importance of testing assets. We show that inferences on relative performance depend on the choice of testing assets. For instance, the presence of both UMD and the monthly formed HML causes difficulties in explaining annually formed value-versus-growth anomalies in the Barillas–Shanken model, difficulties that are absent from the Fama–French five-factor model and the *q*-factor model. As such, it is crucial to use a large set of testing assets to draw reliable inferences. Finally, our evidence on how a given anomaly can be explained by different factor models is important in its own right.

Unlike investment and profitability, expected growth is unobservable. We must take a stand on its empirical specification, such as the list of predictors to be included. We

acknowledge that the expected growth factor depends on the specification, and crucially, on operating cash flows as a predictor. While it is intuitive why operating cash flows are linked to expected growth, we emphasize a minimalistic interpretation of our evidence as empirical dimension reduction. By more than halving the number of anomalies unexplained by the *q*-factor model from 52 to 23, with only one extra factor, the q^5 model makes further progress toward the goal of dimension reduction (Cochrane, 2011).

George, Hwang, and Li (2018) show that the ratio of current price to 52-week high price contains information about future growth, information that helps explain the accrual and R&D-to-market anomalies. Li and Wang (2018) use earnings before extraordinary items and depreciation but after interest and taxes, along with Tobin's *q* and prior 11-month returns, to forecast capital expenditure growth. A rich accounting literature motivates operating cash flows as a key predictor of future growth. Ball et al. (2016) show that cash-based profitability outperforms earnings-based profitability in forecasting returns. Lev (2001) and Lev and Gu (2016) argue that expensing R&D and other intangible investments per current accounting standards make earnings a poor indicator of future growth. Penman (2009) argues that the value of intangibles can be ascertained from variables, such as cash flows, from the income statement. Lev, Radhakrishnan, and Zhang (2009) estimate firm-specific organizational capital from its impact on operating cash flows via revenue growth and cost containment.

The rest of the article is organized as follows. Section 2 motivates the expected growth factor. Section 3 forms cross-sectional growth forecasts and constructs the expected growth factor. Section 4 stress-tests the factor models. Finally, Section 5 concludes. A separate Supplementary Appendix details derivations, factor construction, and additional results.

2. Motivating Expected Growth

Hou, Xue, and Zhang (2015) underpin the *q*-factor model on a static investment framework, which we extend to a dynamic setting to motivate the expected growth factor. Section 2.1 describes the economic model. Section 2.2 presents its implications on cross-sectional returns. Finally, Section 2.3 interprets the factors from the investment theory.

2.1 Conceptual Framework

Time is discrete, and the horizon infinite. The economy is populated by a representative household and heterogeneous firms, indexed by $i = 1, 2, \ldots, N$. The household maximizes its expected utility, $E_0[\sum_{t=0}^{\infty} \rho^t U(C_t)]$, in which ρ is time preference, C_t time t consumption, and $U(\cdot)$ the period utility function. Heterogeneous firms use capital and costlessly adjustable inputs to produce a homogeneous output, which can be consumed or invested. These inputs are chosen each period to maximize operating profits (defined as revenue minus the costs of these inputs). Taking operating profits as given, firms choose investment to maximize their market value of equity.

Let $\Pi_{it} = X_{it} A_{it}$ be firm i's time t operating profits, in which A_{it} is productive assets and X_{it} stochastic return on assets. X_{it} is subject to aggregate and firm-specific shocks. Let I_{it} be investment and δ the depreciation rate of assets, then $A_{it+1} = I_{it} + (1 - \delta) A_{it}$. Changing the scale of assets incurs adjustment costs, which are quadratic, $(a/2)(I_{it}/A_{it})^2 A_{it}$, in which $a > 0$. For simplicity, we assume that the firm has no debt and pays no taxes. The net payout of the firm is $D_{it} = X_{it} A_{it} - I_{it} - (a/2)(I_{it}/A_{it})^2 A_{it}$. If $D_{it} \geq 0$, the firm distributes it to

shareholders. A negative D_{it} means that the firm raises an amount of external equity that equals the absolute value of D_{it}.

The pricing implications of the household's problem are well known. Let P_{it} be the ex-dividend equity and D_{it} dividend of firm i. The consumption-based capital asset pricing model (consumption CAPM) based on the first principle for consumption and portfolio choice says that $E_t[M_{t+1}R_{it+1}^S] = 1$, in which $M_{t+1} \equiv \rho U'(C_{t+1})/U'(C_t)$ is the stochastic discount factor given by the household's intertemporal marginal rate of substitution, and $R_{it+1}^S \equiv (P_{it+1} + D_{it+1})/P_{it}$ is the stock return. Equivalently, $E_t[R_{it+1}^S] - R_{ft} = \beta_{it}^M \lambda_{Mt}$, in which $R_{ft} \equiv 1/E_t[M_{t+1}]$ is the real interest rate, $\beta_{it}^M \equiv -\text{Cov}(R_{it+1}^S, M_{t+1})/\text{Var}(M_{t+1})$ the consumption beta, and $\lambda_{Mt} \equiv \text{Var}(M_{t+1})/E_t[M_{t+1}]$ the price of the consumption risk.

On the production side, taking M_{t+1} as given, firm i chooses the optimal investment stream, $\{I_{is}\}_{s=0}^{\infty}$, to maximize the market equity, $E_0[\sum_{s=0}^{\infty} M_{is}D_{is}]$. The first principle of investment says that $E_t[M_{t+1}R_{it+1}^I] = 1$, in which the investment return, R_{it+1}^I, is:

$$R_{it+1}^I = \frac{X_{it+1} + (a/2)(I_{it+1}/A_{it+1})^2 + (1-\delta)[1 + a(I_{it+1}/A_{it+1})]}{1 + a(I_{it}/A_{it})}. \tag{1}$$

Intuitively, the investment return is the marginal benefit of investment at time $t+1$ divided by the marginal cost of investment at t. $E_t[M_{t+1}R_{it+1}^I] = 1$ says that the marginal cost equals the next period marginal benefit discounted to time t with the stochastic discount factor. In the numerator, X_{it+1} is the marginal profits produced by an extra unit of assets, $(a/2)(I_{it+1}/A_{it+1})^2$ is the marginal reduction in adjustment costs, and the last term is the marginal continuation value of the extra unit of assets, net of depreciation.

Cochrane (1991) uses no-arbitrage to argue, and Restoy and Rockinger (1994) prove under constant returns to scale, that the stock return equals the investment return period by period and state by state (Supplementary Appendix). Equation (1) says that the stock return equals the next period marginal benefit of investment divided by the current marginal cost of investment. Intuitively, the firm will keep investing until the marginal cost of investment, which rises with investment, equals the present value of an extra unit of assets given by the next period marginal benefit of investment discounted by the discount rate (the stock return). With debt and taxes, Liu, Whited, and Zhang (2009) show that the left-hand side of Equation (1) becomes the weighted average cost of capital. As such, the equation is exactly the net present value rule of capital budgeting in corporate finance.

2.2 Pricing Implications

In a static model, in which $I_{it+1} = 0$, Equation (1) collapses to $R_{it+1}^S = (X_{it+1} + 1 - \delta)/(1 + aI_{it}/A_{it})$. All else equal, low-investment stocks should earn higher expected returns than high-investment stocks, and high expected profitability stocks should earn higher expected returns than low expected profitability stocks. Intuitively, given expected profitability, high costs of capital give rise to low net present values of new projects and low investment. Given investment, high expected profitability implies high discount rates, which are necessary to offset the high expected profitability to induce low net present values of new projects to keep investment low. Hou, Xue, and Zhang (2015) build on these insights to form the investment and return on equity factors in the *q*-factor model.

In the multiperiod framework, Equation (1) says that holding investment and expected profitability constant, the expected return also increases with the expected investment-to-assets growth. The right-hand side of Equation (1) can be decomposed into a "dividend

yield" and a "capital gain." The former is $[X_{it+1} + (a/2)(I_{it+1}/A_{it+1})^2]/(1 + aI_{it}/A_{it})$, which largely conforms to the static model, as the squared term, $(I_{it+1}/A_{it+1})^2$, is economically small. The "capital gain," $(1 - \delta)(1 + aI_{it+1}/A_{it+1})/(1 + aI_{it}/A_{it})$, is the growth of marginal q (the market value of an extra unit of assets). Although the "capital gain" involves the unobservable parameter, a, it is roughly proportional to the investment-to-assets growth, $(I_{it+1}/A_{it+1})/(I_{it}/A_{it})$ (Cochrane, 1991). As such, the expected investment-to-assets growth is the third "determinant" of the expected return.

The intuition is exactly analogous to the intuition underlying the positive profitability-expected return relation. The term, $1 + aI_{it+1}/A_{it+1}$, is the marginal cost of investment next period, which, per the first principle of investment, equals the marginal q next period (the present value of cash flows in all future periods arising from one extra unit of assets next period). The expected marginal q is then part of the expected marginal benefit of current investment. This term is absent from the static model that abstracts from growth in subsequent periods. As such, in the multiperiod world, if expected investment is high relative to current investment, the discount rate must be high to offset the high expected marginal benefit of current investment to keep current investment low.

2.3 Interpreting Factors: An Investment Perspective

Hou, Xue, and Zhang (2015) implement the static version of Equation (1) with the Fama–French (1993) portfolio approach. Hou et al. construct factor mimicking portfolios on investment and profitability and use the factors in the right-hand side of time-series regressions. Analogously, we build an expected growth factor to form an expanded factor model. The time-honored portfolio approach takes advantage of high-frequency stock returns data, which are less subject to measurement errors than accounting variables. Structurally estimating Equation (1) directly as in Liu, Whited, and Zhang (2009) involves specification errors in the marginal product of capital and the marginal cost of investment, errors that are largely avoided in the factor approach.

Because the factor approach and structural estimation are two different ways of implementing the investment theory, we interpret the q^5 model as a linear approximation to the firm-level cost of capital given by the nonlinear Equation (1). The equation says that the expected return varies cross-sectionally, depending on firms' investment, expected profitability, and expected investment growth. The q^5 model operationalizes this insight by forming factors on the three "determinants."

As two different ways of summarizing correlations between returns and characteristics, factor models and cross-sectional regressions are largely equivalent on economic grounds. If a characteristic is significant in cross-sectional regressions, its factor likely earns a significant premium, and vice versa. Factor loadings are no more primitive than characteristics, and vice versa, in explaining average returns (Lin and Zhang, 2013).

The return comovements among stocks with similar investment, profitability, and expected growth, in the sense of Ross (1976), can arise from the comovements in their investment returns due to the similar characteristics. In particular, stocks with similar investment-to-assets comove in their stock returns because their investment returns are similar due to similar investment-to-assets in the denominator of Equation (1). Analogously, stocks with similar profitability comove in their stock returns because their investment returns are similar due to similar profitability in the numerator. Finally, stocks

with similar expected investment growth comove in their stock returns because their investment returns are similar due to similar expected investment growth.[1]

In this sense, we echo Kozak, Nagel, and Santosh (2018) in that horse races between covariances and characteristics cannot shed light on the efficient markets versus behavioral finance debate. In their model, distorted beliefs drive the cross-section of expected returns, but the cross-section can be captured empirically by a low-dimensional factor model. While Kozak et al. emphasize that the evidence that a factor model explains returns is not inconsistent with mispricing, we emphasize that the evidence that characteristics explain returns is not inconsistent with efficient markets.

More important, we interpret the q^5 model as summarizing a large amount of the cross-sectional variation in average returns (dimension reduction). This interpretation is distinctively weaker than the risk factors interpretation of Fama and French (1993, 1996). We are keenly aware that our evidence is not inconsistent with mispricing (Lin and Zhang, 2013). For example, the stock market might not adequately value intangibles (Edmans, 2011), giving rise to a positive relation between operating cash flows and average subsequent returns. Future work can shed further light on the economic driving forces behind the investment, profitability, and expected growth factors.

3. Constructing the Expected Growth Factor

We perform cross-sectional growth forecasts in Section 3.1, form the expected growth factor in Section 3.2, and explore alternative growth specifications in Section 3.3.

3.1 Cross-Sectional Growth Forecasts

A technical issue arises in that firm-level investment is frequently negative, making the growth rate of investment-to-assets ill-defined. As such, we forecast future investment-to-assets changes. Forecasting changes captures the essence of the economic insight that all else equal, high expected investment-to-assets relative to current investment-to-assets must imply high discount rates. Our forecasting framework is monthly Fama–MacBeth (1973) cross-sectional predictive regressions. At the beginning of each month t, we measure current investment-to-assets as total assets (Compustat annual item AT) from the most recent fiscal year ending at least four months ago minus the total assets from one year prior, scaled by the 1-year-prior total assets. The left-hand side variables in the cross-sectional regressions are investment-to-assets changes, denoted $d^\tau I/A$, in which $\tau = 1, 2$, and 3 years. We measure $d^1 I/A$, $d^2 I/A$, and $d^3 I/A$ as investment-to-assets from the 1st, 2nd, and 3rd fiscal year after the most recent fiscal year end minus the current investment-to-assets, respectively. The sample is from July 1963 to December 2018.

Following Cooper, Gulen, and Schill (2008), Hou, Xue, and Zhang (2015) measure investment-to-assets as asset growth when constructing the investment factor in their *q*-

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1 This comovement mechanism in the investment theory, which is based on the optimality condition of an individual firm, is microeconomic in nature. In contrast, the consumption CAPM works through a representative consumer, which gives rise to aggregate consumption growth as the key factor. However, the aggregate nature of the factor is a direct consequence of the strong aggregation assumption embedded in the consumption CAPM. In all, characteristics-based factors are on as solid theoretical grounds in the investment theory as aggregation consumption growth in the consumption theory (Zhang, 2017).

factor model. Because our conceptual framework is a dynamic extension of Hou et al.'s static model, we adopt the same definition to be consistent. Fama and French (2015, 2018) also use the same definition of investment. Cooper, Gulen, and Ion (2017) argue that the asset growth premium is mostly driven by the growth in noncash current assets, as opposed to long-term investment in fixed assets. However, the net present value intuition underlying the investment factor applies not only to long-term investment in fixed assets but also to short-term investment in working capital. In the presence of capital heterogeneity, asset growth is a simple, convenient measure of the ratio of total investments to total assets, a measure that summarizes the predictive power of investments on different capital goods.[2]

3.1.a. *Predictors Based on a Priori Conceptual Arguments*

Which variables should one use to forecast future growth? Our goal is a conceptually motivated yet empirically validated specification for the expected investment-to-assets changes. Keynes (1936) and Tobin (1969) argue that a firm should invest if its average q exceeds one. Lucas and Prescott (1971) and Mussa (1977) show that optimality requires the marginal cost of investment to equal marginal q. With quadratic adjustment costs, this first-order condition can be rewritten as a linear regression of investment-to-assets on (unobservable) marginal q. Hayashi (1982) shows that under constant returns to scale, marginal q equals (observable) average q. As such, we include Tobin's q as a predictor.

Cash flows typically have economically large and statistically significant slopes once included in the investment-q regression. Fazzari, Hubbard, and Petersen (1988) show that the cash flows effect on investment is especially strong for firms that are more financially constrained. However, the interpretation of the cash flows effect is controversial.[3] We remain agnostic about the exact interpretation of the cash flows effect, which is not directly related to our asset pricing questions. As such, we also include cash flows on the right-hand side of our forecasting regressions.

More important, a rich accounting literature motivates cash flows as a key predictor of future growth. Ball et al. (2016) document that cash-based profitability outperforms earnings-based profitability in forecasting returns. The evidence suggests that firms with high accruals earn lower average returns because of their lower profitability on a cash basis. We complement their interpretation by linking cash flows and accruals to expected growth. Intuitively, high cash flows mean more internal funds available for investments, giving rise to high expected growth and expected returns. In addition, high accruals mean low cash flows, all else equal, giving rise to low expected growth and expected returns.

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2 Wu, Zhang, and Zhang (2010) use the net present value intuition on working capital to explain the accruals anomaly. Belo and Lin (2012) show that the relation between inventory and average returns arises from a two-capital model. Goncalves, Xue, and Zhang (2019) show that a two-capital model with working and physical capital goods does a good job in fitting the value, momentum, asset growth, and profitability premiums simultaneously via structural estimation. Their main challenge is to explain the value premium, while simultaneously accounting for momentum. The asset growth premium poses no particular difficulty.

3 Using measurement error-consistent estimation, Erickson and Whited (2000) find that cash flows do not matter in the investment-q regression even for financially constrained firms and interpret the cash flows effect as indicative of measurement errors in Tobin's q. In addition, the investment-cash flows relation can arise theoretically even without financial constraints (Gomes, 2001; Alti, 2003; Abel and Eberly, 2011). Finally, in a model with financial constraints, cash flows matter only if one ignores marginal q (Gomes, 2001).

A large national accounting literature shows that intangible investments have become more important than tangible investments as a fraction of gross value added over the past two decades (Corrado, Hulten, and Sichel, 2009). Haskel and Westlake (2018) describe the broad-ranging consequences of the rise of the intangible economy. In financial accounting, intangible investments such as R&D, advertising, supply chains, information technology, and employee training are all immediately expensed, making earnings a poor indicator of expected growth and the market value (Lev, 2001; Lev and Gu, 2016). Because their future payoffs are uncertain, intangible assets fail to meet the criteria for asset recognition in the balance sheet per accounting standards. However, intangibles have become arguably more important than fixed assets in driving expected growth and the market value.

Penman (2009) argues that omitting intangibles from the balance sheet is not necessarily deficient because the value of intangibles can be ascertained from the flow variables in the income statement. For example, although missing from its balance sheet, the brand value of the Coca-Cola Company directly impacts on its operating profits (revenue minus operating costs). In particular, Lev, Radhakrishnan, and Zhang (2009) develop a firm-specific measure of organizational capital based on its impact on operating cash flows by increasing revenues and containing costs. Lev et al. show that their measure correlates positively with future growth in operating profits and sales. These powerful accounting insights motivate operating cash flows as a key predictor of future growth.

Finally, both Tobin's *q* and cash flows are slow-moving. To help capture the short-term dynamics of expected growth, we also include the change in return on equity over the past four quarters, denoted dRoe, as a predictor of growth. Intuitively, firms that experience recent increases in profitability tend to raise future investments in the short term, and firms that experience recent decreases in profitability tend to reduce future investments.[4]

3.1.b. *Measuring Growth Predictors*

Monthly returns are from the Center for Research in Security Prices (CRSP) and accounting information from the Compustat Annual and Quarterly Fundamental Files. We require CRSP share codes to be 10 or 11. Financial firms and firms with negative book equity are excluded. Our measure of Tobin's *q* is standard (Kaplan and Zingales, 1997). At the beginning of each month *t*, current Tobin's *q* is the market equity (price per share times the number of shares outstanding from CRSP) plus long-term debt (Compustat annual item DLTT) and short-term debt (item DLC) scaled by book assets (item AT), all from the most recent fiscal year ending at least four months ago. For firms with multiple share classes, we merge the market equity for all classes.

We follow Ball et al. (2016) in measuring operating cash flows, denoted Cop. At the beginning of each month *t*, we measure current Cop as total revenue (Compustat annual item REVT) minus cost of goods sold (item COGS), minus selling, general, and administrative

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4 Novy-Marx (2015) argues that the investment theory cannot explain momentum measured as dRoe. However, Liu, Whited, and Zhang (2009) show that firms that experience recently positive earnings shocks have higher average future investment growth than firms that experience recently negative earnings shocks. Liu and Zhang (2014) show that this investment growth spread is temporary, converging within 12 months, and helps explain the short duration of momentum. Goncalves, Xue, and Zhang (2019) show that a detailed treatment of aggregation and capital heterogeneity enables the investment theory to explain value and momentum simultaneously. We instead form cross-sectional growth forecasts to build the expected growth factor.

expenses (item XSGA), plus research and development expenditures (item XRD, zero if missing), minus change in accounts receivable (item RECT), minus change in inventory (item INVT), minus change in prepaid expenses (item XPP), plus change in deferred revenue (item DRC plus item DRLT), plus change in trade accounts payable (item AP), and plus change in accrued expenses (item XACC), scaled by book assets, all from the fiscal year ending at least four months ago. Missing annual changes are set to zero.

The change in return on equity, dRoe, is Roe minus the 4-quarter-lagged Roe. Roe is income before extraordinary items (Compustat quarterly item IBQ) scaled by the 1-quarter-lagged book equity. We compute dRoe with earnings from the most recent announcement dates (item RDQ), and if not available, from the fiscal quarter ending at least four months ago (Hou, Xue, and Zhang, 2019). Finally, missing dRoe values are set to zero in the cross-sectional forecasting regressions.

3.1.c. *Cross-Sectional Forecasting Regressions*

Panel A of Table I shows monthly cross-sectional regressions of future investment-to-assets changes on the log of Tobin's q, $\log(q)$; cash flows, Cop; and the change in return on equity, dRoe. We winsorize both the left- and right-hand side variables each month at the 1–99% level. To control for the impact of microcaps, we use weighted least squares with the market equity as the weights.

At the beginning of each month t, we construct expected τ-year-ahead investment-to-assets changes, denoted $E_t[d^\tau I/A]$, in which $\tau = 1, 2$, and 3 years, by combining most recent winsorized predictors with the average slopes estimated from the prior 120-month rolling window (30 months minimum). The most recent predictors, $\log(q)$ and Cop, in $E_t[d^\tau I/A]$ are from the most recent fiscal year ending at least four months ago as of month t, and dRoe is computed using the latest announced quarterly earnings, and if not available, the earnings from the most recent fiscal quarter ending at least four months ago.

The average slopes in calculating $E_t[d^\tau I/A]$ are estimated from the prior rolling window regressions, in which $d^\tau I/A$ is from the most recent fiscal year ending at least four months ago as of month t, and the regressors are further lagged accordingly. For instance, for $\tau = 1$, the regressors in the latest monthly cross-sectional regression are further lagged by 12 months relative to the most recent predictors that we combine with the slopes in calculating $E_t[d^1 I/A]$. Finally, we report the time-series averages of cross-sectional Pearson and rank correlations between $E_t[d^\tau I/A]$ calculated at the beginning of month t and the subsequent τ-year-ahead investment-to-assets changes after month t.

Panel A shows that Tobin's q alone is a weak predictor of investment-to-assets changes. At the 1-year horizon, the slope, 0.02, is small, albeit significant. The R^2 is only 1%, which is not surprising when forecasting changes. The out-of-sample correlations are also close to zero.[5] Operating cash flows, Cop, perform better than Tobin's q. When used alone, Cop has significant slopes that range from 0.42 to 0.46 (t-values above 10). The out-of-sample

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5 Forecasting growth rates often yields low explanatory power. For example, Chan, Karceski, and Lakonishok (2003) document a low R^2 for earnings growth, even with a myriad of predictors, including valuation ratios. Also, in untabulated results, we show that the time-series average of the contemporaneous cross-sectional Pearson correlation between $\log(q)$ and investment-to-assets is 0.23, and the rank correlation 0.3. The investment theory predicts a tight relation of Tobin's q with the current investment level, but not necessarily with future investment-to-assets changes.

Table I. Monthly cross-sectional regressions of future investment-to-assets changes (July 1963 to December 2018)

For each month, we perform cross-sectional regressions of future τ-year-ahead investment-to-assets changes, $d^\tau I/A$, in which $\tau = 1, 2, 3$, on the log of Tobin's q, $\log(q)$; cash flows, Cop; the change in return on equity, dRoe; as well as on all the three regressors. Current investment-to-assets is from the most recent fiscal year ending at least four months ago, and $d^\tau I/A$ is investment-to-assets from the subsequent τ-year-ahead fiscal year end minus the current investment-to-assets. The cross-sectional regressions are estimated via weighted least squares with the market equity as the weights. We winsorize each variable each month at the 1–99% level. We report the average slopes, the *t*-values adjusted for heteroskedasticity and autocorrelations (in parentheses), and goodness-of-fit coefficients, R^2. At the beginning of each month *t*, we calculate the expected I/A changes, $E_t[d^\tau I/A]$, by combining the most recent winsorized predictors with the average cross-sectional slopes. The most recent predictors, $\log(q)$ and Cop, are from the most recent fiscal year ending at least four months ago as of month *t*, and dRoe is based on the latest announced earnings, and if not available, the earnings from the most recent fiscal quarter ending at least four months ago. The average slopes in calculating $E_t[d^\tau I/A]$ are from the prior 120-month rolling window (30 months minimum), in which the dependent variable, $d^\tau I/A$, uses data from the fiscal year ending at least four months ago as of month *t*, and the regressors are further lagged accordingly. For instance, for $\tau = 1$, the regressors used in the latest monthly cross-sectional regression are further lagged by 12 months relative to the most recent predictors used in calculating $E_t[d^1 I/A]$. We report time-series averages of cross-sectional Pearson and rank correlations between $E_t[d^\tau I/A]$ calculated at the beginning of month *t* and the realized τ-year-ahead investment-to-assets changes. The *p*-values testing that a given correlation is zero are in square brackets.

	Panel A: $\log(q)$				Panel B: Cop			
τ	$\log(q)$	R^2	Pearson	Rank	Cop	R^2	Pearson	Rank
1	0.021	0.01	0.016	0.004	0.418	0.03	0.138	0.176
	(5.12)		[0.00]	[0.33]	(13.38)		[0.00]	[0.00]
2	−0.005	0.01	0.027	0.037	0.457	0.04	0.127	0.153
	(−0.95)		[0.00]	[0.00]	(12.09)		[0.00]	[0.00]
3	−0.019	0.01	0.085	0.098	0.436	0.04	0.115	0.131
	(−3.81)		[0.00]	[0.00]	(10.49)		[0.00]	[0.00]

	Panel C: dRoe				Panel D: $\log(q)$, Cop, and dRoe					
τ	dRoe	R^2	Pearson	Rank	$\log(q)$	Cop	dRoe	R^2	Pearson	Rank
1	0.795	0.02	0.068	0.131	−0.029	0.516	0.771	0.06	0.135	0.208
	(7.85)		[0.00]	[0.00]	(−5.63)	(12.75)	(7.62)		[0.00]	[0.00]
2	0.949	0.02	0.068	0.155	−0.073	0.699	0.907	0.09	0.148	0.220
	(9.82)		[0.00]	[0.00]	(−9.76)	(12.34)	(10.07)		[0.00]	[0.00]
3	0.746	0.02	0.055	0.130	−0.093	0.745	0.717	0.09	0.154	0.218
	(8.50)		[0.00]	[0.00]	(−12.39)	(12.17)	(8.60)		[0.00]	[0.00]

correlations are much higher than those with Tobin's q. The change in return on equity, dRoe, performs better than Tobin's q, but not cash flows. When used alone, the dRoe slopes range from 0.75 to 0.95, with t-values above 7.5.

In our benchmark specification with $\log(q)$, Cop, and dRoe together, the slopes are similar to those from univariate regressions. At the 1-year horizon, for instance, the Cop slope remains large and significant, 0.52, the $\log(q)$ slope becomes weakly negative, –0.03, and the dRoe slope stays significant at 0.77. The in-sample R^2 increases to 6.4%. The out-of-sample Pearson and rank correlations, which are important for constructing the expected growth factor, are 0.14 and 0.21, respectively. Both are highly significant. At the 3-year horizon, the $\log(q)$ and Cop slopes both increase in magnitude to –0.09 and 0.75, respectively, but the dRoe slope falls slightly to 0.72. The in-sample R^2 rises to 9%, and the out-of-sample correlations rise slightly to 0.15 and 0.22, respectively.

3.2 The Expected Growth Premium

Armed with the cross-sectional forecasts of investment-to-assets changes, we form the expected growth deciles, construct an expected growth factor, and augment the q-factor model with the new factor to form the q^5 model.

3.2.a. *Deciles*

At the beginning of each month t, we form deciles on the expected investment-to-assets changes, $E_t[\mathrm{d}^\tau\mathrm{I}/\mathrm{A}]$, with $\tau = 1, 2$, and 3 years. As in Table I, we calculate $E_t[\mathrm{d}^\tau\mathrm{I}/\mathrm{A}]$ by combining most recent winsorized predictors with the average slopes from the prior 120-month rolling window (30 months minimum). We sort all stocks into deciles on the NYSE breakpoints of the ranked $E_t[\mathrm{d}^\tau\mathrm{I}/\mathrm{A}]$ values and calculate the value-weighted decile returns for the current month t. The deciles are rebalanced at the beginning of month $t + 1$.

Panel A of Table II shows a reliable expected growth premium. The high-minus-low $E_t[\mathrm{d}^1\mathrm{I}/\mathrm{A}]$ decile earns on average 1.07% per month ($t = 6.48$). The high-minus-low $E_t[\mathrm{d}^2\mathrm{I}/\mathrm{A}]$ and $E_t[\mathrm{d}^3\mathrm{I}/\mathrm{A}]$ deciles earn on average about 1.18%, with t-values above 7. From Panel B, the expected growth premium cannot be explained by the q-factor model. The high-minus-low alphas are 0.86, 0.93, and 1.01% ($t = 6.19, 5.53$, and 6.01) over the 1-, 2-, and 3-year horizons, respectively. The mean absolute alphas across the deciles are 0.23, 0.21, and 0.24%, respectively, and the q-factor model is strongly rejected by the GRS test on the null that the alphas are jointly zero across a given set of deciles (untabulated).

Panel C reports the expected investment-to-assets changes, and Panel D the average subsequently realized changes across the $E_t[\mathrm{d}^\tau\mathrm{I}/\mathrm{A}]$ deciles. Both the expected and realized changes are value-weighted at the portfolio level with the market equity as the weights. Reassuringly, the expected changes track the subsequently realized changes closely. At the 1-year horizon, the expected changes rise monotonically from –15.21% per annum for Decile 1 to 7.65% for Decile 10, and the average realized changes from –16.69% for Decile 1 to 5.96% for Decile 10. As such, our $E_t[\mathrm{d}^\tau\mathrm{I}/\mathrm{A}]$ proxy is close to an unbiased estimator at the portfolio level, which diversifies away firm-level estimation errors. The time-series average of cross-sectional correlations between the expected and realized changes is 0.64, which is highly significant (untabulated). The evidence for the 2- and 3-year horizons is largely similar, with correlations of 0.7 and 0.67, respectively. As such, our empirical specification for the expected investment-to-assets changes seems effective.

Table II. Properties of the expected growth deciles (January 1967 to December 2018)

We use the log of Tobin's q, log(q); cash flow, Cop; and the change in return on equity, dRoe, to form the expected investment-to-assets changes, $E_t[\mathrm{d^\tau I/A}]$, with τ ranging from one to three years. At the beginning of each month t, we calculate $E_t[\mathrm{d^\tau I/A}]$ by combining the three most recent predictors (winsorized at the 1–99% level) with the average slopes. The most recent predictors, log(q) and Cop, are from the most recent fiscal year ending at least as well as months ago as of month t, and dRoe uses the latest announced earnings, and if not available, the earnings from the most recent fiscal quarter ending at least four months ago. The average slopes in calculating $E_t[\mathrm{d^\tau I/A}]$ are from the prior 120-month rolling window (30 months minimum), in which the dependent variable, $\mathrm{d^\tau I/A}$, uses data from the fiscal year ending at least four months ago as of month t, and the regressors are further lagged accordingly. For instance, for $\tau = 1$, the regressors used in the latest monthly cross-sectional regression are further lagged by 12 months relative to the most recent predictors used in calculating $E_t[\mathrm{d^1 I/A}]$. Cross-sectional regressions are estimated via weighted least squares with the market equity as weights. At the beginning of each month t, we sort all stocks into deciles based on the NYSE breakpoints of the ranked $E_t[\mathrm{d^\tau I/A}]$ values, and compute value-weighted decile returns for the current month t. The deciles are rebalanced at the beginning of month $t + 1$. For each decile and the high-minus-low decile, we report the average excess return, $\bar{R}$, the q-factor alpha, α_q, the expected investment-to-assets changes, $E_t[\mathrm{d^\tau I/A}]$, and the average future realized changes, $\mathrm{d^\tau I/A}$, as well as their heteroskedasticity-and-autocorrelation-adjusted t-statistics (beneath the corresponding estimates). $E_t[\mathrm{d^\tau I/A}]$ and $\mathrm{d^\tau I/A}$ are value-weighted.

τ		Low	2	3	4	5	6	7	8	9	High	H–L
					Panel A: Average excess returns, $\bar{R}$							
1	$\bar{R}$	−0.12	0.20	0.28	0.42	0.45	0.49	0.56	0.64	0.77	0.95	1.07
	t	−0.40	0.84	1.21	2.00	2.36	2.61	3.00	3.54	4.17	4.69	6.48
2	$\bar{R}$	−0.09	0.23	0.23	0.37	0.44	0.60	0.62	0.80	0.70	1.08	1.17
	t	−0.33	0.98	1.07	1.79	2.29	3.36	3.50	4.23	3.61	5.10	7.14
3	$\bar{R}$	−0.08	0.20	0.30	0.39	0.53	0.51	0.74	0.68	0.81	1.11	1.19
	t	−0.29	0.90	1.41	1.92	2.82	2.79	3.86	3.39	4.19	5.20	7.13
					Panel B: The q-factor alphas, α_q							
1	α_q	−0.42	−0.35	−0.23	−0.14	−0.15	−0.02	0.08	0.17	0.29	0.43	0.86
	t	−4.09	−3.45	−2.28	−1.58	−1.80	−0.28	1.05	1.64	3.54	4.31	6.19
2	α_q	−0.36	−0.19	−0.17	−0.19	−0.13	0.06	0.01	0.17	0.29	0.58	0.93
	t	−3.78	−2.43	−1.81	−2.88	−1.81	0.68	0.19	1.88	3.02	4.16	5.53
3	α_q	−0.40	−0.16	−0.21	−0.23	−0.02	−0.11	0.17	0.19	0.30	0.61	1.01
	t	−4.14	−1.84	−2.49	−3.00	−0.21	−1.21	1.88	1.98	3.02	4.40	6.01
					Panel C: The expected growth, $E_t[\mathrm{d^\tau I/A}]$							
1	$E_t[\mathrm{d^\tau I/A}]$	−15.21	−7.67	−5.61	−4.20	−3.03	−1.97	−0.86	0.47	2.52	7.65	22.87
	t	−36.75	−31.37	−25.19	−20.56	−15.96	−11.01	−5.08	3.01	16.53	37.98	45.21
2	$E_t[\mathrm{d^\tau I/A}]$	−19.87	−10.18	−7.38	−5.52	−4.03	−2.67	−1.23	0.51	3.13	9.44	29.31
	t	−34.26	−26.34	−21.16	−16.88	−12.97	−8.94	−4.22	1.81	11.30	29.57	45.51
3	$E_t[\mathrm{d^\tau I/A}]$	−20.42	−11.16	−8.26	−6.33	−4.75	−3.31	−1.77	0.03	2.66	9.06	29.48
	t	−30.59	−23.07	−18.58	−15.04	−11.80	−8.51	−4.70	0.10	7.67	24.92	44.17

Table II. Continued

τ		Low	2	3	4	5	6	7	8	9	High	H–L
					Panel D: Average future realized growth, d^{τ}I/A							
1	d^{τ}I/A	−16.69	−12.30	−4.11	−3.56	−1.10	−0.43	−0.32	0.64	1.57	5.96	22.65
	t	−11.71	−8.36	−7.15	−5.22	−2.24	−0.90	−0.71	1.18	3.59	9.07	14.72
2	d^{τ}I/A	−23.68	−12.64	−6.45	−3.74	−2.25	−1.44	0.10	1.47	1.25	3.14	26.82
	t	−14.38	−12.42	−8.44	−4.60	−3.86	−2.43	0.22	2.72	2.33	4.93	16.10
3	d^{τ}I/A	−23.10	−12.91	−7.00	−3.20	−2.29	−2.90	−1.44	−0.50	0.46	1.31	24.41
	t	−14.70	−13.87	−9.51	−4.72	−3.79	−4.68	−2.96	−0.91	0.76	1.85	15.18

3.2.b. *A Common Factor*

In view of the expected growth premium largely unexplained by the *q*-factor model, we form an expected growth factor, denoted R_{Eg}. Following the standard factor construction procedure per Fama and French (1993), we form R_{Eg} from an independent 2 × 3 sort on the market equity and the expected 1-year-ahead investment-to-assets changes, $E_t[d^1 I/A]$.

At the beginning of each month *t*, we use the beginning-of-month median NYSE market equity to split stocks into two groups, small and big. Independently, we split all stocks into three groups, low, medium, and high, based on the NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked $E_t[d^1 I/A]$ values. Taking the intersection of the two sizes and three $E_t[d^1 I/A]$ groups, we form six benchmark portfolios. Monthly value-weighted portfolio returns are calculated for the current month *t*, and the portfolios are rebalanced at the beginning of month $t + 1$. Designed to mimic the common variation related to $E_t[d^1 I/A]$, the expected growth factor, R_{Eg}, is the difference (high-minus-low), each month, between the simple average of the returns on the two high $E_t[d^1 I/A]$ portfolios and the simple average of the returns on the two low $E_t[d^1 I/A]$ portfolios.

Panel A of Table III shows the basic properties of the expected growth factor, R_{Eg}. In the 1967–2018 sample, its average return is 0.84% per month ($t = 10.27$). The *t*-value clears the high hurdle of three that adjusts for multiple testing per Harvey, Liu, and Zhu (2016). The *q*-factor alpha of R_{Eg} is large, 0.67% ($t = 9.75$). Its investment and return on equity factor loadings are both significantly positive, 0.21 and 0.3 ($t = 4.86$ and 9.13), respectively, yet still leave the bulk of the average return unexplained. As such, the expected growth factor captures a new dimension of the expected return variation missed by the *q*-factor model. In untabulated results, the Fama–French (2018) six-factor alpha of the expected growth factor is 0.71% ($t = 11.71$). Based on the six-factor model, Chordia, Goyal, and Saretto (2019) propose a *t*-value cutoff of 3.84 for alphas to control for multiple testing. Our *t*-value of 11.71 far exceeds this high hurdle.

The subsequent five regressions in Panel A identify the sources behind the expected growth premium. To this end, we form factors on log(*q*), Cop, and dRoe, by interacting each of them separately with the market equity in 2 × 3 sorts. Cop is the most important component of the expected growth premium. Augmenting the *q*-factor model with the Cop factor reduces the alpha of R_{Eg} from 0.67% per month ($t = 9.75$) to 0.37% ($t = 6.35$). dRoe plays a more limited role. Adding the dRoe factor to the *q*-factor model reduces the alpha only slightly to 0.63% ($t = 8.56$). Tobin's *q* is negligible on its own but more effective when

Table III. Properties of the expected growth factor, R_{Eg} (January 1967 to December 2018)

The log of Tobin's *q*, $\log(q)$; cash flows, Cop; and change in return on equity, dRoe, are used to form the expected 1-year-ahead investment-to-assets changes, $E_t[d^1I/A]$. At the beginning of month *t*, $E_t[d^1I/A]$ combines the most recent predictors (winsorized at the 1–99% level) with average Fama–MacBeth slopes. The most recent $\log(q)$ and Cop are from the most recent fiscal year ending at least four months ago as of month *t*, and dRoe uses the latest announced earnings, and if not available, the earnings from the most recent fiscal quarter ending at least four months ago. The average slopes in calculating $E_t[d^\tau I/A]$ are from the prior 120-month rolling window (30 months minimum), in which the dependent variable, d^1I/A, uses data from the fiscal year ending at least four months ago as of month *t*, and the regressors are further lagged. We estimate the regressions via weighted least squares with the market equity as weights. At the beginning of each month *t*, we use the median NYSE market equity to split stocks into two groups, small and big, based on the beginning-of-month market equity. Independently, we sort all stocks into three $E_t[d^1I/A]$ groups, low, median, and high, based on the NYSE breakpoints for the low 30%, middle 40%, and high 30% of its ranked values at the beginning of month *t*. Taking the intersections, we form six portfolios. We calculate value-weighted portfolio returns for the current month *t*, and rebalance the portfolios at the beginning of month *t* + 1. The expected growth factor, R_{Eg}, is the difference (high-minus-low), each month, between the simple average of the returns on the two high $E_t[d^1I/A]$ portfolios and the simple average of the returns on the two low $E_t[d^1I/A]$ portfolios. Panel A reports for the expected growth factor, R_{Eg}, its average return, $\bar{R}_{Eg}$, and alphas, factor loadings, and R^2 from the *q*-factor model, and the *q*-factor model augmented with a $\log(q)$ factor, a Cop factor, and a dRoe factor, separately or jointly. The *t*-values adjusted for heteroskedasticity and autocorrelations are in parentheses. To form the $\log(q)$ and Cop factors, we use independent annual sorts (with size) at the end of June of year *t*, with NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked values from the fiscal year ending in calendar year *t* − 1. To form the dRoe factor, we use independent monthly sorts (with size) at the beginning of each month *t*, with NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked values of dRoe. dRoe is calculated with the latest announced earnings, and if not available, with the earnings from the fiscal quarter ending at least four months ago. Panel B reports the correlations of the expected growth factor, R_{Eg}, with the *q*-factors, as well as the $\log(q)$, Cop, and dRoe factors.

Panel A: Properties of the expected growth factor, R_{Eg}							
$\bar{R}_{Eg}$	α	β_{Mkt}	β_{Me}	$\beta_{I/A}$	β_{Roe}	R^2	
0.84	0.67	−0.11	−0.09	0.21	0.30	0.44	
(10.27)	(9.75)	(−6.38)	(−3.56)	(4.86)	(9.13)		
	α	β_{Mkt}	β_{Me}	$\beta_{I/A}$	β_{Roe}	$\beta_{\log(q)}$	R^2
	0.67	−0.11	−0.09	0.23	0.30	−0.02	0.44
	(9.80)	(−6.40)	(−3.61)	(4.72)	(8.83)	(−0.48)	
	α	β_{Mkt}	β_{Me}	$\beta_{I/A}$	β_{Roe}	β_{Cop}	R^2
	0.37	−0.02	−0.02	0.31	0.14	0.60	0.65
	(6.35)	(−1.66)	(−0.54)	(9.51)	(4.37)	(10.63)	

(continued)

Table III. Continued

				Panel A: Properties of the expected growth factor, R_{Eg}			
α	β_{Mkt}	β_{Me}	$\beta_{\text{I/A}}$	β_{Roe}	β_{dRoe}	R^2	
0.63	−0.11	−0.10	0.18	0.23	0.16	0.46	
(8.56)	(−6.62)	(−3.93)	(3.57)	(5.00)	(2.41)		

α	β_{Mkt}	β_{Me}	$\beta_{\text{I/A}}$	β_{Roe}	β_{Cop}	β_{dRoe}	R^2
0.33	−0.03	−0.02	0.28	0.07	0.60	0.15	0.66
(5.20)	(−1.88)	(−0.72)	(6.73)	(1.72)	(10.02)	(2.33)	

α	β_{Mkt}	β_{Me}	$\beta_{\text{I/A}}$	β_{Roe}	$\beta_{\log(q)}$	β_{Cop}	β_{dRoe}	R^2
0.25	−0.01	−0.01	0.06	0.04	0.22	0.72	0.21	0.70
(4.04)	(−0.86)	(−0.35)	(1.31)	(1.27)	(8.36)	(14.61)	(3.19)	

			Panel B: Correlations of R_{Eg} with other factors			
R_{Mkt}	R_{Me}	$R_{\text{I/A}}$	R_{Roe}	$R_{\log(q)}$	R_{Cop}	R_{dRoe}
−0.458	−0.367	0.342	0.506	0.188	0.710	0.423

used together with Cop and dRoe. Adding the $\log(q)$, Cop, and dRoe factors together to the q-factor model yields an alpha of 0.25% ($t = 4.04$), which is lower than 0.33% ($t = 5.2$) when adding only the Cop and dRoe factors.[6]

Panel B shows that the expected growth factor has positive correlations of 0.34 and 0.51 with the investment and Roe factors but negative correlations of −0.46 and −0.37 with the market and size factors in the q-factor model. All are significant from zero.

3.2.c. The q^5 Model

We augment the q-factor model with the expected growth factor to form the q^5 model. The expected excess return of an asset, denoted $E[R_i - R_f]$, is described by the loadings of its returns to five factors, including the market factor, R_{Mkt}; the size factor, R_{Me}; the investment factor, $R_{\text{I/A}}$; the return on equity factor, R_{Roe}; and the expected growth factor, R_{Eg}. The first four factors are identical to those in the q-factor model. Formally,

$$E[R_i - R_f] = \beta_{\text{Mkt}}^i E[R_{\text{Mkt}}] + \beta_{\text{Me}}^i E[R_{\text{Me}}] + \beta_{\text{I/A}}^i E[R_{\text{I/A}}] + \beta_{\text{Roe}}^i E[R_{\text{Roe}}] + \beta_{\text{Eg}}^i E[R_{\text{Eg}}], \quad (2)$$

in which $E[R_{\text{Mkt}}], E[R_{\text{Me}}], E[R_{\text{I/A}}], E[R_{\text{Roe}}]$, and $E[R_{\text{Eg}}]$ are the expected factor premiums, and $\beta_{\text{Mkt}}^i, \beta_{\text{Me}}^i, \beta_{\text{I/A}}^i, \beta_{\text{Roe}}^i$, and β_{Eg}^i are their factor loadings, respectively.

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6 We form the log (q) and Cop factors with annual sorts to facilitate comparison with the existing literature (Ball et al. 2016). In untabulated results, we have also examined the log (q) and Cop factors with monthly sorts that are analogous to our construction of the expected growth factor. Tobin's q continues to play a negligible role when used alone. Adding the monthly sorted Cop factor into the q-factor model yields an alpha of 0.27% ($t = 5.16$) for the expected growth factor, and adding all three monthly formed factors reduces the alpha further to 0.16% ($t = 2.9$).

Not surprisingly, the expected growth factor explains the deciles sorted on the expected 1-year-ahead investment-to-assets changes, $E_t[\mathrm{d}^1\mathrm{I/A}]$ (Supplementary Appendix). The high-minus-low decile earns a q^5 alpha of only -0.15% per month ($t = -1.5$) due to a large expected growth factor loading of 1.5 ($t = 26.75$). The mean absolute alpha is only 0.07%, and the GRS test cannot reject the q^5 model ($p = 0.13$). Reassuringly, the expected growth factor also explains the $E_t[\mathrm{d}^2\mathrm{I/A}]$ and $E_t[\mathrm{d}^3\mathrm{I/A}]$ deciles. The high-minus-low alphas are only -0.05% ($t = -0.43$) and 0.05 ($t = 0.38$), the mean absolute alphas 0.07% and 0.09%, and the GRS p-values 0.49 and 0.12, respectively.

3.3 Limitations

Unlike investment and profitability, expected growth is unobservable. Estimating expected growth requires us to take a stand on its specification and the list of predictors to be included. While the t-value of the expected growth factor far exceeds the existing hurdles of multiple testing, the factor depends on its specification, and crucially on operating cash flows, Cop, as a predictor of future growth.[7] While we do provide strong intuition on why cash flows should be linked to future growth (and reliable evidence on this linkage), we emphasize a minimalistic interpretation of our extensive evidence on factor models as dimension reduction. In particular, among the 52 (out of 150) anomalies that the q-factor model cannot explain, cash flows seem to be an important, missing factor.

To what extent do our cross-sectional growth forecasts add to a mechanical combination of the three predictors? We form an alternative expected growth factor on the composite score that equal-weights a stock's percentile rankings of $\log(q)$, Cop, and dRoe (each realigned to yield a positive slope in forecasting returns). The alternative expected growth factor earns on average 0.86% per month ($t = 9.37$), and its q-factor alpha is 0.45% ($t = 6.33$) (Supplementary Appendix). The correlation between the alternative and benchmark expected growth factors is far from perfect, 0.63. The benchmark q^5 model subsumes the alternative expected growth factor, with an alpha of 0.12% ($t = 1.75$), but the alternative q^5 model cannot subsume the benchmark expected growth factor, with an alpha of 0.48% ($t = 6.4$). As such, our growth forecasts capture valuable pricing information beyond the simple, mechanical rule of equal-weighting.

The expected growth factor is robust to changes in the left-hand side variable of the cross-sectional forecasting regressions. As noted, because firm-level investment-to-assets (I/A, net asset growth) is frequently negative, we forecast future investment-to-assets changes, $\mathrm{d}^\tau\mathrm{I/A}$, for $\tau = 1, 2$, and 3 years. We have explored the alternative of forecasting the log growth rate of gross asset growth, denoted $\mathrm{dlog}^\tau(1+\mathrm{I/A})$. The results are largely similar (Supplementary Appendix). In particular, the alternative expected growth factor earns on average 0.84% per month ($t = 10.24$), with a q-factor alpha of 0.67% ($t = 9.62$). Its correlation with the benchmark expected growth factor is 0.99.

The expected growth factor is also relatively robust to changes in the right-hand side variables. We start by adding past investment growth to the right-hand side. Adding 1-year-lagged investment-to-assets changes, $\mathrm{d}^{-1}\mathrm{I/A}$, does not affect the results (Supplementary Appendix). Its slope in the forecasting regression of $\mathrm{d}^1\mathrm{I/A}$ is weakly

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7   This aspect is not that different from the influential stock market predictability literature, in which the predictive results depend on the predictors employed (Welch and Goyal, 2008), as well as the conditional asset pricing literature, in which the pricing results depend on the variables in the conditional beta specifications (Ghysels, 1998).

negative. The resulting expected growth factor earns on average 0.82% per month ($t = 10.35$), with a q-factor alpha of 0.71% ($t = 9.38$). Adding 2-quarter-lagged year-to-year investment-to-assets changes, $d^{-1/2}$I/A, yields a negative slope of –0.15 in the forecasting regression, and the expected growth premium falls slightly to 0.76% ($t = 10.43$), with a q-factor alpha of 0.64% ($t = 9.93$). Adding 1-quarter-lagged year-to-year investment-to-assets changes, $d^{-1/4}$I/A, raises (in magnitude) the slope further to –0.22, and the expected growth premium still remains at 0.71% ($t = 9.93$), with a q-factor alpha of 0.61% ($t = 8.73$). However, if we include current investment-to-assets changes, d^0I/A, the slope rises to –0.45, and the expected growth premium falls to 0.47% ($t = 7.02$), with a q-factor alpha of 0.44% ($t = 5.92$). The large slope is driven by a mechanical relation in the regression because current investment-to-assets appears on both left- and right-hand sides.[8]

Barro (1990) and Morck, Shleifer, and Vishny (1990) propose two alternative investment growth specifications. Barro uses lagged investment growth, 1-year ex-dividend stock market return, the first difference of the ratio of after-tax corporate profits to sales, the log growth of Tobin's q, and the growth of gross national product to forecast aggregate investment growth. Morck et al. regress firm-level capital expenditure growth contemporaneously on the growth of earnings before depreciation, sales growth, new share dummy, new debt dummy, and lagged market regression residuals. We add these variables into our expected growth specification to evaluate its sensitivity. We drop the $\log(q)$ growth from Barro to avoid multicollinearity because $\log(q)$ is already in our specification. We lag all the variables from Morck et al. to avoid look-ahead bias in our forecasting regressions.

Adding Barro's (1990) variables (with 1-year-lagged investment-to-assets changes, d^{-1}I/A) yields an expected growth premium of 0.59% per month ($t = 5.52$), with a q-factor alpha of 0.31% ($t = 1.99$) (Supplementary Appendix). Adding the variables from Morck, Shleifer, and Vishny (1990) into our specification yields an expected growth premium of 0.63% ($t = 7.07$), with a q-factor alpha of 0.43% ($t = 3.35$). To reiterate, our expected growth factor depends on its specification and the list of predictors to be included. In particular, the factor depends crucially on operating cash flows as a key predictor of future growth. While the underlying intuition based on the growing importance of intangible investments seems clear and the t-value of 10.27 for the factor premium exceeds multiple testing hurdles, we emphasize the minimalistic interpretation of dimension reduction.[9]

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8   Imposing a time lag between a dependent variable and its lagged value in a forecasting regression to avoid any mechanical relation is standard in empirical finance. For instance, in the monthly sorts of momentum portfolios, it is standard to impose a 1-month lag between prior and subsequent returns to avoid the short-term reversal due to market microstructure frictions (Jegadeesh and Titman, 1993). As such, we emphasize the robustness when adding 1-year-lagged investment-to-assets changes, d^{-1}I/A, to the forecasting regression.

9   We echo Fama and French (2018) when adding the momentum factor, UMD, into their six-factor specification: "We include momentum factors (somewhat reluctantly) now to satisfy insistent popular demand. We worry, however, that opening the game to factors that seem empirically robust but lack theoretical motivation has a destructive downside: the end of discipline that produces parsimonious models and the beginning of a dark age of data dredging that produces a long list of factors with little hope of sifting through them in a statistically reliable way" (p. 237).

4. Stress-Testing Factor Models

The most demanding test of the q^5 model is to confront it with a vast set of testing anomaly portfolios. We also conduct a large-scale empirical horse race with other competing factor models. We set up the playing field in Section 4.1, discuss the overall performance of different factor models in Section 4.2, and detail individual regressions for prominent anomalies in Section 4.3.

4.1 The Playing Field

We describe testing portfolios as well as different factor models in the empirical horse race.

4.1.a. *Testing Portfolios*

We use the 150 anomalies that are significant at the 5% level with NYSE breakpoints and value-weighted returns from January 1967 to December 2018 (Hou, Xue, and Zhang, 2019). Table IV provides the detailed list, which includes 39, 15, 26, 40, 27, and 3 across the momentum, value-versus-growth, investment, profitability, intangibles, and trading frictions categories, respectively.[10]

The list contains 52 anomalies that cannot be explained by the *q*-factor model. Prominent examples include cumulative abnormal stock returns around quarterly earnings announcement dates (Chan, Jegadeesh, and Lakonishok, 1996), customer momentum (Cohen and Frazzini, 2008), and segment momentum (Cohen and Lou, 2012) in the momentum category; net payout yield (Boudoukh et al., 2007) in the value-versus-growth category; operating accruals (Sloan, 1996), discretionary accruals (Xie, 2001), net operating assets (Hirshleifer et al., 2004), and net stock issues (Pontiff and Woodgate, 2008) in the investment category; asset turnover (Soliman, 2008) and operating profits-to-assets (Ball et al., 2015) in the profitability category; R&D-to-market (Chan, Lakonishok, and Sougiannis, 2001) and seasonalities (Heston and Sadka, 2008) in the intangibles category.

4.1.b. *Factor Models*

In addition to the *q* and q^5 models, we examine six other models, including (i) the Fama–French (2015) five-factor model; (ii) the Fama–French (2018) six-factor model with RMW; (iii) the Fama–French alternative six-factor model with RMWc; (iv) the Barillas–Shanken (2018) six-factor model; (v) the Stambaugh–Yuan (2017) four-factor model; and (vi) the Daniel–Hirshleifer–Sun (2019) three-factor model. As shown in Hou et al. (2019), the explanatory power of the Stambaugh–Yuan and Daniel et al. models is exaggerated because both deviate from the standard factor contribution per Fama and French (1993). To level the playing field, we use the replicated versions of the two models per the standard construction. Supplementary Appendix describes the replication of these factor models in detail.

Table V reports monthly Sharpe ratios for individual factors and maximum Sharpe ratios for different factor models. The maximum Sharpe ratio for a given factor model is calculated as $\sqrt{\mu_f' V_f^{-1} \mu_f}$, in which μ_f is the vector of mean factor returns, and V_f the variance–covariance matrix of the factor returns in the model (MacKinlay, 1995). From Panel

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10 In their original 1967–2016 sample, Hou, Xue, and Zhang (2019) report 158 significant anomalies, including 36, 29, 28, 35, 26, and 4 across the momentum, value-versus-growth, investment, profitability, intangibles, and trading frictions categories, respectively. We extend the sample through December 2018. The big news is in the value-versus-growth category, in which the number of significance drops drastically from 29 to 15. The number of significance increases slightly in the momentum and profitability categories but stays largely the same in the other three categories.

Table IV. The list of significant anomalies to be explained

The 150 anomalies (significant with NYSE breakpoints and value-weighted returns) are grouped into six categories: (i) momentum; (ii) value-versus-growth; (iii) investment; (iv) profitability; (v) intangibles; and (vi) trading frictions. The number in parenthesis in the title of a panel is the number of anomalies in that category. For each anomaly variable, we list its symbol, brief description, and its academic source. Hou, Xue, and Zhang (2019) detail variable definition and portfolio construction.

	Panel A: Momentum (39)		
Sue1	Earnings surprise (1-month period), Foster, Olsen, and Shevlin (1984)	Abr1	Cumulative abnormal returns around earnings announcements (1-month period), Chan, Jegadeesh, and Lakonishok (1996)
Abr6	Cumulative abnormal returns around earnings announcements (6-month period), Chan, Jegadeesh, and Lakonishok (1996)	Abr12	Cumulative abnormal returns around earnings announcements (12-month period), Chan, Jegadeesh, and Lakonishok (1996)
Re1	Revisions in analysts' forecasts (1-month period), Chan, Jegadeesh, and Lakonishok (1996)	Re6	Revisions in analysts' forecasts (6-month period), Chan, Jegadeesh, and Lakonishok (1996)
$R^6 1$	Price momentum (6-month prior returns, 1-month period), Jegadeesh and Titman (1993)	$R^6 6$	Price momentum (6-month prior returns, 6-month period), Jegadeesh and Titman (1993)
$R^6 12$	Price momentum (6-month prior returns, 12-month period), Jegadeesh and Titman (1993)	$R^{11} 1$	Price momentum (11-month prior returns, 1-month period), Fama and French (1996)
$R^{11} 6$	Price momentum, (11-month prior returns, 6-month period), Fama and French (1996)	$R^{11} 12$	Price momentum, (11-month prior returns, 12-month period), Fama and French (1996)
Im1	Industry momentum (1-month period), Moskowitz and Grinblatt (1999)	Im6	Industry momentum (6-month period), Moskowitz and Grinblatt (1999)
Im12	Industry momentum (12-month period), Moskowitz and Grinblatt (1999)	Rs1	Revenue surprise (1-month period), Jegadeesh and Livnat (2006)
dEf1	Analysts' forecast change (1-month period), Hawkins, Chamberlin, and Daniel (1984)	dEf6	Analysts' forecast change (6-month period), Hawkins, Chamberlin, and Daniel (1984)
dEf12	Analysts' forecast change (12-month period), Hawkins, Chamberlin, and Daniel (1984)	Nei1	# of consecutive quarters with earnings increases (1-month period), Barth, Elliott, and Finn (1999)
52w6	52-week high (6-month period), George and Hwang (2004)	52w12	52-week high (12-month period), George and Hwang (2004)
$\epsilon^6 6$	6-month residual momentum (6-month period), Blitz, Huij, and Martens (2011)	$\epsilon^6 12$	6-month residual momentum (12-month period), Blitz, Huij, and Martens (2011)
$\epsilon^{11} 1$	11-month residual momentum (1-month period), Blitz, Huij, and Martens (2011)	$\epsilon^{11} 6$	11-month residual momentum (6-month period), Blitz, Huij, and Martens (2011)
$\epsilon^{11} 12$	11-month residual momentum (12-month period), Blitz, Huij, and Martens (2011)	Sm1	Segment momentum (1-month period), Cohen and Lou (2012)

(continued)

Table IV. Continued

Sm12	Segment momentum (12-month period), Cohen and Lou (2012)	Ilr1	Industry lead-lag effect in prior returns (1-month period), Hou (2007)
Ilr6	Industry lead-lag effect in prior returns (6-month period), Hou (2007)	Ilr12	Industry lead-lag effect in prior returns (12-month period), Hou (2007)
Ile1	Industry lead-lag effect in earnings news (1-month period), Hou (2007)	Cm1	Customer momentum (1-month period), Cohen and Frazzini (2008)
Cm12	Customer momentum (12-month period), Cohen and Frazzini (2008)	Sim1	Supplier industries momentum (1-month period), Menzly and Ozbas (2010)
Cim1	Customer industries momentum (1-month period), Menzly and Ozbas (2010)	Cim6	Customer industries momentum (6-month period), Menzly and Ozbas (2010)
Cim12	Customer industries momentum (12-month period), Menzly and Ozbas (2010)		

Panel B: Value-versus-growth (15)

Bm	Book-to-market equity, Rosenberg, Reid, and Lanstein (1985)	Ep^q1	Quarterly earnings-to-price (1-month period)
Ep^q6	Quarterly earnings-to-price (6-month period)	Ep^q12	Quarterly earnings-to-price (12-month period)
Cp^q1	Quarterly cash flow-to-price (1-month period)	Cp^q6	Quarterly cash flow-to-price (6-month period)
Nop	Net payout yield, Boudoukh et al. (2007)	Em	Enterprise multiple, Loughran and Wellman (2011)
Em^q1	Quarterly enterprise multiple (1-month period)	Sp	Sales-to-price, Barbee, Mukherji, and Raines (1996)
Sp^q1	Quarterly sales-to-price (1-month period)	Sp^q6	Quarterly sales-to-price (6-month period)
Sp^q12	Quarterly sales-to-price (12-month period)	Ocp	Operating cash flow-to-price, Desai, Rajgopal, and Venkatachalam (2004)
Ocp^q1	Quarterly operating cash flow-to-price (1-month period)		

Panel C: Investment (26)

Ia	Investment-to-assets, Cooper, Gulen, and Schill (2008)	Ia^q6	Quarterly investment-to-assets (6-month period)
Ia^q12	Quarterly investment-to-assets (12-month period)	dPia	(Changes in PPE and inventory)/assets, Lyandres, Sun, and Zhang (2008)
Noa	Net operating assets, Hirshleifer et al. (2004)	dNoa	Changes in net operating assets, Hirshleifer et al. (2004)
dLno	Change in long-term net operating assets, Fairfield, Whisenant, and Yohn (2003)	Ig	Investment growth, Xing (2008)
2Ig	Two-year investment growth, Anderson and Garcia-Feijoo (2006)	Nsi	Net stock issues, Pontiff and Woodgate (2008)

(continued)

Table IV. Continued

dIi	% change in investment–% change in industry investment, Abarbanell and Bushee (1998)	Cei	Composite equity issuance, Daniel and Titman (2006)
Ivg	Inventory growth, Belo and Lin (2012)	Ivc	Inventory changes, Thomas and Zhang (2002)
Oa	Operating accruals, Sloan (1996)	dWc	Change in net noncash working capital, Richardson et al. (2005)
dCoa	Change in current operating assets, Richardson et al. (2005)	dNco	Change in net noncurrent operating assets, Richardson et al. (2005)
dNca	Change in noncurrent operating assets, Richardson et al. (2005)	dFin	Change in net financial assets, Richardson et al. (2005)
dFnl	Change in financial liabilities, Richardson et al. (2005)	dBe	Change in common equity, Richardson et al. (2005)
Dac	Discretionary accruals, Xie (2001)	Poa	Percent operating accruals, Hafzalla, Lundholm, and Van Winkle (2011)
Pta	Percent total accruals, Hafzalla, Lundholm, and Van Winkle (2011)	Pda	Percent discretionary accruals

Panel D: Profitability (40)

Roe1	Return on equity (1-month period), Hou, Xue, and Zhang (2015)	Roe6	Return on equity (6-month period), Hou, Xue, and Zhang (2015)
dRoe1	Change in Roe (1-month period)	dRoe6	Change in Roe (6-month period)
dRoe12	Change in Roe (12-month period),	Roa1	Return on assets (1-month period), Balakrishnan, Bartov, and Faurel (2010)
dRoa1	Change in Roa (1-month period)	dRoa6	Change in Roa (6-month period)
Ato	Asset turnover, Soliman (2008)	Cto	Capital turnover, Haugen and Baker (1996)
Rna^q1	Quarterly return on net operating assets (1-month period)	Rna^q6	Quarterly return on net operating assets (6-month period)
Ato^q1	Quarterly asset turnover (1-month period)	Ato^q6	Quarterly asset turnover (6-month period)
Ato^q12	Quarterly asset turnover (12-month period)	Cto^q1	Quarterly capital turnover (1-month period)
Cto^q6	Quarterly capital turnover (6-month period)	Cto^q12	Quarterly capital turnover (12-month period)
Gpa	Gross profits-to-assets, Novy-Marx (2013)	Gla^q1	Gross profits-to-lagged assets (1-month period)
Gla^q6	Gross profits-to-lagged assets (6-month period)	Gla^q12	Gross profits-to-lagged assets (12-month period)
Ole^q1	Operating profits-to-lagged equity (1-month period)	Ole^q6	Operating profits-to-lagged equity (6-month period)
Opa	Operating profits-to-assets, Ball et al. (2015)	Ola^q1	Operating profits-to-lagged assets (1-month period)
Ola^q6	Operating profits-to-lagged assets (6-month period)	Ola^q12	Operating profits-to-lagged assets (12-month period)
Cop	Cash-based operating profitability, Ball et al. (2016)	Cla	Cash-based operating profits-to-lagged assets

(continued)

Table IV. Continued

Cla^{q}1	Cash-based operating profits-to-lagged assets (1-month period)	Cla^{q}6	Cash-based operating profits-to-lagged assets (6-month period)
Cla^{q}12	Cash-based operating profits-to-lagged assets (12-month period)	F^{q}1	Quarterly F-score (1-month period)
F^{q}6	Quarterly F-score (6-month period)	F^{q}12	Quarterly F-score (12-month period)
Fp^{q}6	Failure probability (6-month period), Campbell, Hilscher, and Szilagyi (2008)	O^{q}1	Quarterly O-score (1-month period)
Tbi^{q}12	Quarterly taxable income-to-book income (12-month period)	Sg^{q}1	Quarterly sales growth (1-month period)

Panel E: Intangibles (27)

Oca	Organizational capital/assets, Eisfeldt and Papanikolaou (2013)	Ioca	Industry-adjusted organizational capital/assets, Eisfeldt and Papanikolaou (2013)
Adm	Advertising expense-to-market, Chan, Lakonishok, and Sougiannis (2001)	Rdm	R&D-to-market, Chan, Lakonishok, and Sougiannis (2001)
Rdm^{q}1	Quarterly R&D-to-market (1-month period)	Rdm^{q}6	Quarterly R&D-to-market (6-month period)
Rdm^{q}12	Quarterly R&D-to-market (12-month period)	Rds^{q}6	Quarterly R&D-to-sales (6-month period)
Rds^{q}12	Quarterly R&D-to-sales (12-month period)	Ol	Operating leverage, Novy-Marx (2011)
Ol^{q}1	Quarterly operating leverage (1-month period)	Ol^{q}6	Quarterly operating leverage (6-month period)
Ol^{q}12	Quarterly operating leverage (12-month period)	Hs	Industry concentration (sales), Hou and Robinson (2006)
Rer	Real estate ratio, Tuzel (2010)	Eprd	Earnings predictability, Francis et al. (2004)
Etl	Earnings timeliness, Francis et al. (2004)	Alm^{q}1	Quarterly market assets liquidity (1-month period)
Alm^{q}6	Quarterly market assets liquidity (6-month period)	Alm^{q}12	Quarterly market assets liquidity (12-month period)
R_a^1	Year 1-lagged return, annual Heston and Sadka (2008)	R_n^1	Year 1-lagged return, nonannual Heston and Sadka (2008)
$R_a^{[2,5]}$	Years 2–5 lagged returns, annual Heston and Sadka (2008)	$R_a^{[6,10]}$	Years 6–10 lagged returns, annual Heston and Sadka (2008)
$R_n^{[6,10]}$	Years 6–10 lagged returns, nonannual Heston and Sadka (2008)	$R_a^{[11,15]}$	Years 11–15 lagged returns, annual Heston and Sadka (2008)
$R_a^{[16,20]}$	Years 16–20 lagged returns, annual Heston and Sadka (2008)		

Panel F: Trading frictions (3)

Dtv12	Dollar trading volume (12-month period), Brennan, Chordia, and Subrahmanyam (1998)	Isff1	Idiosyncratic skewness per the 3-factor model (1-month period)
Isq1	Idiosyncratic skewness per the *q*-factor model (1-month period)		

A, the individual Sharpe ratio is the highest, 0.44, for the expected growth factor, R_{Eg}, followed by the post-earnings-announcement-drift (PEAD) factor, 0.32. The investment factor, $R_{I/A}$, has a Sharpe ratio of 0.2, which is higher than 0.15 for CMA. The Roe factor, R_{Roe}, has a Sharpe ratio of 0.22, which is higher than 0.13 for RMW and 0.19 for RMWc.

Panel B shows that the q^5 model has the highest maximum Sharpe ratio, 0.63, among all the factor models. The Sharpe ratio for the q-factor model is 0.42, which compares favorably with 0.37 for the Fama–French (2018) six-factor model, but falls slightly short of 0.43 for their alternative six-factor model. The Barillas–Shanken (2018) six-factor model has a higher Sharpe ratio of 0.48 than the q-factor model. Based on this evidence, Barillas and Shanken argue that their six-factor model is a better model than the q-factor model and testing assets are largely irrelevant. Our extensive evidence based on 150 anomalies casts doubt on their conclusion (Sections 4.2 and 4.3).

4.2 The Big Picture of Model Performance

4.2.a. *Performance across 150 Anomalies*

Panel A of Table VI shows the overall performance of different factor models in explaining the 150 significant anomalies. The q^5 model is the overall best performer. The q-factor model performs well too, with a lower number of significant high-minus-low alphas but a higher number of rejections by the GRS test than the Fama–French six-factor model and the Stambaugh–Yuan model. The Fama–French five-factor, the Barillas–Shanken, and the Daniel–Hirshleifer–Sun models all perform poorly.

The q-factor model leaves 52 significant high-minus-low alphas with $|t| \geq 1.96$ and 25 with $|t| \geq 3$. The average magnitude of high-minus-low alphas is 0.28% per month. Across all 150 sets of deciles, the mean absolute alpha is 0.11%, but the q-factor model is still rejected by the GRS test at the 5% level in 101 sets of deciles. The q^5 model improves on the q-factor model substantially. The average magnitude of high-minus-low alphas is 0.19% per month. The number of significant high-minus-low alphas is 23 with $|t| \geq 1.96$ and 6 with $|t| \geq 3$, dropping from 52 and 25, respectively, in the q-factor model. The mean absolute alpha across all the deciles is 0.1%. Finally, the q^5 model is rejected by the GRS test in only 57 sets of deciles, and this number of GRS rejections represents a reduction of 44% from 101 in the q-factor model.

The Fama–French five-factor model performs poorly. The model leaves 100 high-minus-low alphas with $|t| \geq 1.96$ and 69 with $|t| \geq 3$. Both are the highest across all factor models. The average magnitude of high-minus-low alphas is 0.43% per month. The model is rejected by the GRS test in 112 sets of deciles. The Fama–French six-factor model performs better. The numbers of high-minus-low alphas with $|t| \geq 1.96$ and $|t| \geq 3$ fall to 74 and 37, respectively. The average magnitude of high-minus-low alphas drops to 0.3%, and the number of GRS rejections to 91. However, other than the lower number of GRS rejections, the six-factor model underperforms the q-factor model.

Replacing RMW with RMWc in the Fama–French six-factor model improves its performance. The average magnitude of high-minus-low alphas falls to 0.27% per month, which is on par with the q-factor model. The number of significant high-minus-low alphas with $|t| \geq 1.96$ drops to 59, which is still higher than 52 in the q-factor model. Finally, the number of GRS rejections falls to 71, which is substantially lower than 101 in the q-factor model but still higher than 57 in the q^5 model. The q^5 model also outperforms the alternative six-factor model with RMWc in all the other metrics.

Table V. Monthly Sharpe ratios (January 1967 to December 2018)

Panel A reports Sharpe ratios for the market, size, investment, and Roe factors in the Hou–Xue–Zhang *q*-factor model (*q*), R_{Mkt}, R_{Me}, $R_{I/A}$, and R_{Roe}, respectively; the expected growth factor, R_{Eg}, in the q^5 model (q^5); the size, value, conservative-minus-aggressive investment, and profitability factors in the Fama–French five-factor model (FF5), SMB, HML, CMA, and RMW, respectively; UMD in the Fama–French six-factor model (FF6); the cash-based profitability factor, RMWc, in the Fama–French alternative six-factor model (FF6c); the monthly formed value factor, HML^m, in the Barillas–Shanken six-factor model (BS6); the management (MGMT) and performance (PERF) factors in the Stambaugh–Yuan four-factor model (SY4); and the financing (FIN) and PEAD factors in the Daniel–Hirshleifer–Sun three-factor model (DHS). Panel B reports the maximum Sharpe ratios for each factor model, calculated as $\sqrt{\mu_f' V_f^{-1} \mu_f}$, in which μ_f is the vector of mean factor returns in the factor model, and V_f is the variance–covariance matrix for the vector of factor returns.

Panel A: Sharpe ratios for individual factors							
R_{Mkt}	R_{Me}	$R_{I/A}$	R_{Roe}	R_{Eg}	SMB	HML	CMA
0.112	0.094	0.200	0.218	0.444	0.074	0.112	0.149
RMW	RMWc	UMD	HML^m	MGMT	PERF	FIN	PEAD
0.125	0.186	0.151	0.083	0.195	0.163	0.104	0.320

Panel B: Maximum Sharpe ratios for factor models							
q	q^5	FF5	FF6	FF6c	BS6	SY4	DHS
0.416	0.634	0.322	0.365	0.434	0.475	0.412	0.416

The Barillas–Shanken six-factor model performs poorly. The average magnitude of high-minus-low alphas is 0.29% per month. The numbers of significant high-minus-low alphas with $|t| \geq 1.96$ and $|t| \geq 3$ are 63 and 37, respectively. The mean absolute alpha across all deciles is 0.13%. The number of GRS rejections is 132 (out of 150). This number of rejections is the highest among all factor models.

The Stambaugh–Yuan four-factor model performs well. It underperforms the *q*-factor model in terms of the number of high-minus-low alphas with $|t| \geq 1.96$ (64 versus 52) but outperforms in the number of GRS rejections (87 versus 101). However, the q^5 model substantially outperforms the Stambaugh–Yuan model in all metrics.

Finally, the Daniel–Hirshleifer–Sun three-factor model performs poorly. The average magnitude of high-minus-low alphas is 0.37% per month, which is the second highest among all factor models. The numbers of significant high-minus-low alphas with $|t| \geq 1.96$ and $|t| \geq 3$ are 70 and 33, respectively. The mean absolute alpha across all 150 sets of deciles is 0.14%, which is the highest among all models. The number of GRS rejections is 97.[11]

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11 Supplementary Appendix shows that an alternative Daniel–Hirshleifer–Sun model with the PEAD factor based on Abr only still underperforms the *q*-factor and q^5 models from July 1972 to December 2018. The average magnitude of high-minus-low alphas is 0.32% per month (0.28% in the *q*-factor model and 0.2% in the q^5 model in the same sample), the number of high-minus-low alphas with $|t| \geq 1.96$ is 59 (49 in *q* and 23 in q^5), the number of high-minus-low alphas with $|t| \geq 3$ is 13 (23 in *q* and 5 in q^5), the mean absolute alpha 0.12% (0.12% in *q* and 0.1% in q^5), and the number of GRS rejections 67 (87 in *q* and 53 in q^5).

Table VI. Overall performance of factor models (January 1967 to December 2018)

For each model, $|\overline{\alpha_{H-L}}|$ is the average magnitude of the high-minus-low alphas, $\#|t|\geq1.96$ the number of the high-minus-low alphas with $|t|\geq1.96$, $\#|t|\geq3$ the number of the high-minus-low alphas with $|t|\geq3$, $|\alpha|$ the mean absolute alpha across the anomaly deciles in a given category, and $\#p<5\%$ the number of sets of deciles within a given category, with which the factor model is rejected by the GRS test at the 5% level. We report the results for the q-factor model (q), the q^5 model (q^5), the Fama–French (2015) five-factor model (FF5), the Fama–French (2018) six-factor model with RMW (FF6), the Fama–French alternative six-factor model with RMWc (FF6c), the Barillas–Shanken (2018) six-factor model (BS6), the Stambaugh–Yuan (2017) four-factor model (SY4), and the Daniel–Hirshleifer–Sun (2019) three-factor model (DHS).

	Panel A: All (150)					Panel B: Momentum (39)					Panel C: Value-versus-growth (15)					Panel D: Investment (26)				
	$|\overline{\alpha_{H-L}}|$	$\#|t|\geq1.96$	$\#|t|\geq3$	$|\alpha|$	$\#p<5\%$	$|\overline{\alpha_{H-L}}|$	$\#|t|\geq1.96$	$\#|t|\geq3$	$|\alpha|$	$\#p<5\%$	$|\overline{\alpha_{H-L}}|$	$\#|t|\geq1.96$	$\#|t|\geq3$	$|\alpha|$	$\#p<5\%$	$|\overline{\alpha_{H-L}}|$	$\#|t|\geq1.96$	$\#|t|\geq3$	$|\alpha|$	$\#p<5\%$
q	0.28	52	25	0.11	101	0.25	11	3	0.10	24	0.21	1	0	0.11	8	0.22	9	4	0.10	19
q^5	0.19	23	6	0.10	57	0.17	4	1	0.09	15	0.22	3	0	0.13	7	0.10	1	0	0.08	6
FF5	0.43	100	69	0.13	112	0.62	37	29	0.15	36	0.15	2	0	0.10	7	0.24	10	7	0.09	17
FF6	0.30	74	37	0.11	91	0.27	19	6	0.10	21	0.19	4	0	0.10	9	0.22	10	6	0.09	16
FF6c	0.27	59	25	0.11	71	0.24	14	5	0.09	18	0.17	3	0	0.10	6	0.18	8	2	0.08	7
BS6	0.29	63	37	0.13	132	0.23	12	4	0.12	33	0.23	6	2	0.13	14	0.22	8	6	0.11	24
SY4	0.29	64	25	0.11	87	0.32	19	6	0.10	23	0.24	4	1	0.12	9	0.19	8	3	0.09	17
DHS	0.37	70	33	0.14	97	0.25	10	3	0.14	26	0.78	15	13	0.23	15	0.34	20	4	0.10	22

	Panel E: Profitability (40)					Panel F: Intangibles (27)					Panel G: Trading frictions (3)				
	$|\overline{\alpha_{H-L}}|$	$\#|t|\geq1.96$	$\#|t|\geq3$	$|\alpha|$	$\#p<5\%$	$|\overline{\alpha_{H-L}}|$	$\#|t|\geq1.96$	$\#|t|\geq3$	$|\alpha|$	$\#p<5\%$	$|\overline{\alpha_{H-L}}|$	$\#|t|\geq1.96$	$\#|t|\geq3$	$|\alpha|$	$\#p<5\%$
q	0.25	16	6	0.10	28	0.47	13	11	0.18	19	0.24	2	1	0.10	3
q^5	0.14	5	1	0.09	14	0.36	8	4	0.15	13	0.19	2	0	0.08	2
FF5	0.43	32	23	0.12	32	0.50	17	9	0.16	18	0.22	2	1	0.07	2
FF6	0.31	26	13	0.10	25	0.48	13	11	0.17	18	0.20	2	1	0.07	2
FF6c	0.26	18	7	0.10	21	0.50	14	11	0.17	18	0.20	2	0	0.07	1
BS6	0.31	20	12	0.12	37	0.49	15	11	0.20	21	0.23	2	2	0.09	3
SY4	0.29	20	9	0.10	24	0.38	11	6	0.15	12	0.18	2	0	0.09	2
DHS	0.18	6	1	0.09	13	0.60	16	10	0.19	18	0.50	3	2	0.18	3

4.2.b. *Performance across Each Category of Anomalies*
Panels B–G of Table VI show that the q^5 model improves on the *q*-factor model across most categories, especially in the investment and profitability categories.

Momentum. From Panel B of Table VI, the improvement in the momentum category is noteworthy. Across the 39 significant momentum anomalies, the average magnitude of high-minus-low q^5 alphas is 0.17% per month (0.25% in the *q*-factor model). The q^5 model reduces the number of significant high-minus-low alphas with $|t| \geq 1.96$ from 11 to 4 (3 to 1 with $|t| \geq 3$), the mean absolute alpha from 0.1% per month slightly to 0.09%, and the number of GRS rejections from 24 to 15.

The Fama–French five-factor model shows no explanatory power for momentum, leaving 37 out of 39 high-minus-low alphas with $|t| \geq 1.96$ (29 with $|t| \geq 3$) as well as the GRS rejections in 36 sets of deciles. The average magnitude of high-minus-low alphas, 0.62% per month, and the mean absolute alpha across all deciles, 0.15%, are the highest among all factor models. Even with UMD, the Fama–French six-factor model still leaves 19 high-minus-low alphas significant with $|t| \geq 1.96$ and 6 with $|t| \geq 3$. The six-factor model is rejected by the GRS test in 21 sets of deciles. Changing RMW to RMWc in the Fama–French six-factor model improves the metrics to 14, 5, and 18, respectively. However, the alternative six-factor model still underperforms the q^5 model in all metrics, including the number of GRS rejections (18 versus 15) and the number of significant high-minus-low alphas (14 versus 4 with $|t| \geq 1.96$ and 5 versus 1 with $|t| \geq 3$).

Other than the slightly lower average magnitude of high-minus-low alphas, 0.23% versus 0.25% per month, the Barillas–Shanken six-factor model underperforms the *q*-factor model. The numbers of high-minus-low alphas with $|t| \geq 1.96$ and $|t| \geq 3$ are 12 and 4, in contrast to 11 and 3 in the *q*-factor model, respectively. The mean absolute alpha is 0.12%, and the number of GRS rejections 33. Both are higher than 0.1% and 24 in the *q*-factor model, respectively. The Stambaugh–Yuan four-factor model performs poorly, leaving 19 high-minus-low alphas with $|t| \geq 1.96$ and 6 with $|t| \geq 3$. The average magnitude of high-minus-low alphas is 0.32% (0.25% in the *q*-factor model). Finally, the Daniel–Hirshleifer–Sun three-factor model underperforms the *q*-factor model with a higher mean absolute alpha of 0.14% and a higher number of GRS rejections of 26. However, its number of significant high-minus-low alphas with $|t| \geq 1.96$ is slightly lower at 10.

Value-versus-growth. Panel C of Table VI shows that among the 15 value-versus-growth anomalies, the role of the expected growth factor is limited. The *q*-factor model leaves 1 high-minus-low alphas with $|t| \geq 1.96$ (3 in the q^5 model) and 0 with $|t| \geq 3$ (0 in the q^5 model). The average magnitude of high-minus-low alphas is 0.21% per month, the mean absolute alpha 0.11%, and the number of GRS rejections is 8, compared with 0.22%, 0.13%, and 7 in the q^5 model, respectively.

The Fama–French five-factor model performs very well in this category. The average magnitude of high-minus-low alphas is 0.15% per month, the number of high-minus-low alphas with $|t| \geq 1.96$ is only 2 (0 with $|t| \geq 3$), the mean absolute alpha 0.1%, and the number of GRS rejections 7. This performance benefits from having both CMA and HML, while giving up on momentum. Including UMD per the six-factor model raises the average magnitude of high-minus-low alphas to 0.19%, the number of alphas with $|t| \geq 1.96$ to 4, and the number of GRS rejections to 9. Adopting RMWc in the six-factor model improves these metrics slightly to 0.17%, 3, and 6, respectively.

The Barillas–Shanken six-factor model performs poorly. The average magnitude of high-minus-low alphas is 0.23% per month, the numbers of alphas with $|t| \geq 1.96$ and $|t| \geq 3$ are 6 and 2, respectively, and the mean absolute alpha is 0.13%. More important, the number of GRS rejections is 14 (out of 15 anomalies). Relative to the q-factor model, the Stambaugh–Yuan four-factor model yields higher numbers of significant high-minus-low alphas, 4 with $|t| \geq 1.96$ and 1 with $|t| \geq 3$ (1 and 0 in the q-factor model), and a higher number of GRS rejections, 9 (8 in the q-factor model).

Finally, the Daniel–Hirshleifer–Sun three-factor model performs very poorly. The average magnitude of the high-minus-low alphas is the highest among all models, 0.78% per month. All 15 high-minus-low alphas are significant with $|t| \geq 1.96$ (13 with $|t| \geq 3$). All 15 sets of deciles yield rejections in the GRS test. The mean absolute alpha of 0.23% is also the highest among all models. Intuitively, the value-minus-growth deciles tend to have large and negative PEAD factor loadings, going in the wrong direction in explaining average returns, as well as positive but smaller FIN factor loadings, going in the right direction (untabulated). Because the PEAD premium is larger than the FIN premium, the Daniel–Hirshleifer–Sun model exacerbates the value-versus-growth anomalies.

Investment. Panel D of Table VI shows that the q^5 model is the best performer in the investment category. All but one of the 26 high-minus-low alphas have $|t| \geq 1.96$, and none have $|t| \geq 3$. The number of GRS rejections is 6. The average magnitude of high-minus-low alphas is 0.1% per month, and the mean absolute alpha 0.08%. This performance improves substantially on the q-factor model, which leaves 9 high-minus-low alphas with $|t| \geq 1.96$ and 4 with $|t| \geq 3$, as well as 19 GRS rejections.

The Fama–French six-factor model is largely comparable with the q-factor model. While outperforming the q-factor model, the alternative six-factor model with RMWc underperforms the q^5 model, leaving 8 high-minus-low alphas with $|t| \geq 1.96$ (1 in q^5) and 2 with $|t| \geq 3$ (0 in q^5) as well as 7 GRS rejections (6 in q^5). The average magnitude of high-minus-low alphas is 0.18% (0.1% in q^5).

The Barillas–Shanken six-factor model is comparable with the q-factor model, with a slightly lower number of high-minus-low alphas with $|t| \geq 1.96$ (8 versus 9), but a higher number of GRS rejections (24 versus 19). The Stambaugh–Yuan four-factor model outperforms the q-factor model slightly but underperforms the q^5 model substantially. The average absolute high-minus-low alpha is 0.19% (0.1% in q^5), the number of high-minus-low alphas with $|t| \geq 1.96$ is 8 (1 in q^5), and the number of GRS rejections is 17 (6 in q^5). Finally, the Daniel–Hirshleifer–Sun three-factor model performs the worst, with the highest average magnitude of high-minus-low alphas, 0.34%, the highest number of high-minus-low alphas with $|t| \geq 1.96$, 20, and the second highest number of GRS rejections, 22.

Profitability. From Panel E of Table VI, the q^5 model is the best performer in the profitability category. The model leaves 5 out of 40 high-minus-low alphas with $|t| \geq 1.96$ (16 in the q-factor model) and 1 with $|t| \geq 3$ (6 in q). The average absolute high-minus-low alpha is 0.14% per month (0.25% in q), the mean absolute alpha 0.09% (0.10% in q), and the number of GRS rejections 14 (28 in q).

The other factor models underperform the q^5 model, often substantially. The Fama–French alternative six-factor model with RMWc has a higher number of GRS rejections, 21, a higher average absolute high-minus-low alpha, 0.26%, as well as higher numbers of high-minus-low alphas with $|t| \geq 1.96$, 18, and with $|t| \geq 3$, 7, than the q^5 model. The six-

factor model with RMW performs worse than the alternative six-factor model. The Barillas–Shanken six-factor model underperforms the *q*-factor model in all metrics. Also, other than fewer GRS rejections (24 versus 28), the Stambaugh–Yuan four-factor model also underperforms the *q*-factor model. The Daniel–Hirshleifer–Sun three-factor model outperforms the *q*-factor model, with a lower average magnitude of high-minus-low alphas, 0.18%, a lower number of high-minus-low alphas with $|t| \geq 1.96$, 6, and a lower number of GRS rejections, 13. However, even this performance is weaker than that of the q^5 model.

Intangibles and Trading Frictions. Panel F shows that the q^5 model is the best performer in the intangibles category. The model leaves 8 out of 27 high-minus-low alphas with $|t| \geq 1.96$ (4 with $|t| \geq 3$). The average magnitude of high-minus-low alphas is 0.36% per month, the mean absolute alpha 0.15%, and the number of GRS rejections 13. The second-best performer is the Stambaugh–Yuan model, with only slightly worse metrics than the q^5 model. The *q*-factor model leaves 13 high-minus-low alphas with $|t| \geq 1.96$ and 11 with $|t| \geq 3$. The average magnitude of high-minus-low alphas is 0.47%, the mean absolute alpha 0.18%, and the number of GRS rejections 19. The Fama–French and Barillas–Shanken models deliver largely similar performance as the *q*-factor model. The Daniel–Hirshleifer–Sun model again performs poorly, with the highest average absolute high-minus-low alpha, 0.6%, and the second highest number of high-minus-low alphas with $|t| \geq 1.96$, 16. From Panel G, with only three trading frictions anomalies, the performance of all models is largely similar, except for the Daniel–Hirshleifer–Sun model, which has the highest average magnitude of high-minus-low alphas, 0.5% per month, and the highest mean absolute alpha, 0.18%. The q^5 model leaves 2 high-minus-low alphas with $|t| \geq 1.96$ but 0 with $|t| \geq 3$. The average magnitude of high-minus-low alphas is 0.19%, the mean absolute alpha 0.08%, and the number of GRS rejections 2.

4.2.c. *Testing Deciles Formed on Composite Scores*

As an alternative way to summarize the overall performance of different factor models, we form composite scores across all 150 anomalies as well as across each of the six categories of anomalies. We then use deciles formed on the composite scores as testing portfolios in factor regressions. Although containing less disaggregated information than Table VI, this approach directly quantifies to what extent a given category (as well as all) of anomalies can be explained by a given factor model.

For a given set of anomalies, we construct its composite score for a stock by equal-weighting the stock's percentile rankings for the anomalies in question. Because anomalies forecast returns with different signs, we realign the anomalies to yield positive slopes in forecasting returns before forming the composite score. At the beginning of month *t*, we split stocks into deciles based on the NYSE breakpoints of the composite score that aggregates a given set of anomalies.[12] We calculate value-weighted decile returns for month *t* and rebalance the deciles at the beginning of month $t + 1$.

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12 As detailed in Hou, Xue, and Zhang (2019), some anomaly deciles are formed monthly, whereas others are formed annually. When calculating the percentile rankings for a given anomaly at the beginning of month *t*, we adopt the same sorting frequency as in individual anomaly deciles. The percentile rankings for monthly sorted anomalies are recalculated monthly, and those for annually sorted anomalies are recalculated at the end of each June.

Table VII shows that the q^5 model is the overall best performer. Aggregating all 150 anomalies, the high-minus-low decile is on average 1.69% per month ($t = 9.62$). The high-minus-low alpha is the lowest in the q^5 model, 0.37%, albeit significant ($t = 2.62$). The high-minus-low decile has large, positive loadings on the investment, Roe, and expected growth factors, 0.57, 0.81, and 0.74 ($t = 6.28$, 8.48, and 7.81), respectively. The mean absolute alpha across all deciles is also the lowest in the q^5 model, 0.1%, but the model is still rejected by the GRS test ($p = 0.01$). For the q-factor model, the high-minus-low alpha is 0.86% ($t = 5.64$), and the mean absolute alpha 0.16%. For comparison, the Fama–French six-factor alpha for the high-minus-low decile is 0.94% ($t = 7.46$), and the alternative six-factor alpha with RMWc is 0.82% ($t = 6.77$). The mean absolute alphas are 0.16% and 0.14%, respectively. Both are rejected by the GRS test ($p = 0.00$).

The high-minus-low composite momentum decile earns on average 1.09% per month ($t = 4.21$). The q^5 model yields an insignificant high-minus-low alpha of –0.25% ($t = -0.85$). Both the Roe and expected growth factors contribute to this performance, with large, positive loadings of 1.16 and 0.9 ($t = 5.44$ and 4.49), respectively. The mean absolute alpha is 0.1%, and the q^5 model is not rejected by the GRS test ($p = 0.35$). The q-factor model yields a high-minus-low alpha of 0.35% ($t = 1.04$), the mean absolute alpha of 0.1%, and a GRS p-value of 0.08. For comparison, the Fama–French six-factor model yields a high-minus-low alpha of 0.33% ($t = 2.08$), a mean absolute alpha of 0.09%, and a GRS p-value of 0.06. The alternative six-factor model with RMWc yields a high-minus-low alpha of 0.29% ($t = 1.82$), a mean absolute alpha of 0.1%, and a GRS p-value of 0.04.

The Fama–French six-factor model does a better job than the q^5 model in explaining the composite value-minus-growth premium, which is on average 0.7% per month ($t = 3.47$). The q^5 model yields a high-minus-low alpha of 0.38% ($t = 2.14$), a mean absolute alpha of 0.16%, and a GRS p-value of 0.00. The q-factor model produces a high-minus-low alpha of 0.28% ($t = 1.48$), a mean absolute alpha of 0.13%, and a GRS p-value of 0.00. For comparison, the six-factor model produces a high-minus-low alpha of 0.19% ($t = 1.58$) and a mean absolute alpha of 0.1%, but the model is also rejected by the GRS test ($p = 0.00$). The performance of the alternative six-factor model with RMWc is largely similar. The Fama–French five-factor model is the best performer in this category, with a tiny high-minus-low alpha of 0.04% ($t = 0.3$), albeit still rejected by the GRS test ($p = 0.00$).

The high-minus-low composite investment decile earns on average 0.66% per month ($t = 4.44$). The q^5 model is the best performer, yielding a tiny high-minus-low alpha of 0.06% ($t = 0.54$), a mean absolute alpha of 0.06%, and a GRS p-value of 0.15. The q-factor model yields a high-minus-low alpha of 0.25% ($t = 2.61$), a mean absolute alpha of 0.1%, and a GRS p-value of 0.00. For comparison, the Fama–French six-factor model produces a high-minus-low alpha of 0.27% ($t = 2.84$), a mean absolute alpha of 0.07%, and a GRS p-value of 0.01. The performance of the alternative six-factor model with RMWc is largely similar, except for a GRS p-value of 0.06.

The high-minus-low composite profitability decile earns on average 0.8% per month ($t = 4.64$). The q^5 model performs very well, with a high-minus-low alpha of –0.14% ($t = -1.21$), a mean absolute alpha of 0.08%, and a GRS p-value of 0.09. The q-factor model yields a high-minus-low alpha of 0.28% ($t = 2.31$), a mean absolute alpha of 0.07%, and a GRS p-value of 0.01. For comparison, the Fama–French six-factor model produces a high-minus-low alpha of 0.43% ($t = 3.94$), a mean absolute alpha of 0.09%, and a GRS p-value of 0.00. The alternative six-factor model with RMWc improves the high-minus-low

Table VII. Explaining composite anomalies (January 1967 to December 2018)

We form composite scores across all the 150 anomalies (All) and across each category of anomalies, including momentum (Mom), value-versus-growth (VvG), investment (Inv), profitability (Prof), intangibles (Intan), and trading frictions (Fric). For a given set of anomalies, we construct the composite score by equal-weighting a stock's percentile ranking for each anomaly (realigned to yield a positive slope in forecasting returns). At the beginning of each month t, we split stocks into deciles based on the NYSE breakpoints of the composite scores, and calculate value-weighted returns for month t. The deciles are rebalanced at the beginning of month $t+1$. For each model and each set of deciles, we report the high-minus-low alpha (Panel A), its t-value (Panel B), the mean absolute alpha (Panel C), and the GRS p-value (Panel D). We report the results for the Hou–Xue–Zhang q-factor model (q), the q^5 model (q^5), the Fama–French five-factor model (FF5), the Fama–French six-factor model (FF6), the Fama–French alternative six-factor model with RMWc (FF6c), the Barillas–Shanken six-factor model (BS6), the Stambaugh–Yuan model (SY4), and the Daniel–Hirshleifer–Sun model (DHS). For the q^5 model, Panel E shows the loadings on the market, size, investment, Roe, and expected growth factors (β_{Mkt}, β_{Me}, $\beta_{\mathrm{I/A}}$, β_{Roe}, and β_{Eg}, respectively) and their t-values. The t-values are adjusted for heteroskedasticity and autocorrelations.

		All	Mom	VvG	Inv	Prof	Intan	Fric		All	Mom	VvG	Inv	Prof	Intan	Fric
$\bar{R}$	$t_{\bar{R}}$	1.69	1.09	0.70	0.66	0.80	0.94	0.23	$t_{\bar{R}}$	9.62	4.21	3.47	4.44	4.64	5.27	1.77

	Panel A: The high-minus-low alpha, $\alpha_{\mathrm{H-L}}$								Panel B: $t_{\mathrm{H-L}}$							
q	0.86	0.35	0.28	0.25	0.28	0.42	0.16		5.64	1.04	1.48	2.61	2.31	2.62	1.80
q^5	0.37	−0.25	0.38	0.06	−0.14	0.50	0.15		2.62	−0.85	2.14	0.54	−1.21	3.19	1.60
FF5	1.33	1.21	0.04	0.29	0.60	0.43	0.14		7.94	3.74	0.30	3.11	5.35	3.24	1.80
FF6	0.94	0.33	0.19	0.27	0.43	0.54	0.12		7.46	2.08	1.58	2.84	3.94	4.25	1.53
FF6c	0.82	0.29	0.12	0.27	0.30	0.57	0.12		6.77	1.82	1.05	2.62	2.30	4.17	1.34
BS6	0.68	0.21	−0.16	0.18	0.34	0.26	0.14		4.85	1.26	−1.17	1.73	2.61	1.85	1.60
SY4	0.90	0.43	0.34	0.10	0.37	0.46	0.13		7.61	1.93	2.20	1.00	2.86	3.16	1.50
DHS	0.74	−0.36	0.98	0.55	−0.09	0.89	0.57		4.98	−1.49	5.34	3.83	−0.56	5.24	4.29

	Panel C: The mean absolute alpha, $\overline{|\alpha|}$								Panel D: The GRS p-value, p_{GRS}							
q	0.16	0.10	0.13	0.10	0.07	0.18	0.10		0.00	0.08	0.00	0.00	0.01	0.00	0.00
q^5	0.10	0.10	0.16	0.06	0.08	0.19	0.08		0.01	0.35	0.00	0.15	0.09	0.00	0.06
FF5	0.25	0.27	0.11	0.08	0.12	0.18	0.08		0.00	0.00	0.00	0.00	0.00	0.00	0.05
FF6	0.16	0.09	0.10	0.07	0.09	0.20	0.07		0.00	0.06	0.00	0.01	0.00	0.00	0.07
FF6c	0.14	0.10	0.10	0.06	0.07	0.21	0.06		0.00	0.04	0.00	0.06	0.09	0.00	0.28
BS6	0.13	0.09	0.12	0.09	0.09	0.15	0.11		0.00	0.07	0.00	0.00	0.00	0.00	0.00
SY4	0.16	0.10	0.14	0.07	0.09	0.18	0.09		0.00	0.01	0.00	0.01	0.00	0.00	0.01
DHS	0.14	0.16	0.31	0.12	0.07	0.28	0.13		0.00	0.00	0.00	0.00	0.35	0.00	0.00

	Panel E: The q^5 factor loadings														
β_{Mkt}	−0.03	−0.10	0.06	−0.03	0.03	−0.04	−0.05	t_{Mkt}	−0.63	−1.24	1.16	−1.04	0.93	−0.83	−2.17
β_{Me}	0.21	0.29	0.30	−0.01	−0.03	0.39	0.77	t_{Me}	3.56	1.49	2.27	−0.28	−0.59	3.33	24.50
$\beta_{\mathrm{I/A}}$	0.57	−0.19	1.31	1.24	−0.43	0.69	−0.04	$t_{\mathrm{I/A}}$	6.28	−0.74	9.51	20.32	−5.34	5.13	−0.73
β_{Roe}	0.81	1.16	−0.30	−0.17	1.05	0.35	−0.21	t_{Roe}	8.48	5.44	−2.48	−2.57	15.42	3.17	−5.00
β_{Eg}	0.74	0.90	−0.15	0.29	0.63	−0.11	0.02	t_{Eg}	7.81	4.49	−1.08	4.15	7.63	−0.93	0.28

alpha to 0.3% ($t = 2.3$), the mean absolute alpha to 0.07%, and the GRS p-value to 0.09. Finally, the Daniel–Hirshleifer–Sun model is comparable with the q^5 model.

The high-minus-low composite intangibles decile earns on average 0.94% per month ($t = 5.27$). The q^5 model yields a high-minus-low alpha of 0.5% ($t = 3.19$), a mean absolute alpha of 0.19%, and a GRS p-value of 0.00. The q-factor model has a slightly lower high-minus-low alpha of 0.42% ($t = 2.62$). The Fama–French six-factor model has a somewhat larger high-minus-low alpha, 0.54% ($t = 4.25$), but is otherwise comparable with the q^5 model. Finally, the high-minus-low composite frictions decile only earns an insignificant average return of 0.23% ($t = 1.77$).

4.3 Individual Factor Regressions

To dig deeper, we present individual regressions of the 52 anomalies that the q-factor model cannot explain. To save space, Table VIII reports the alphas and t-values for the q-factor model, the q^5 model, and the two versions of the Fama–French six-factor model, as well as the q^5 loadings for each high-minus-low decile. Supplementary Appendix contains the results for all the 150 anomalies and for all the factor models.

All models including the q and q^5 models fail to explain the anomaly on cumulative abnormal returns around earnings announcements, Abr, especially at the 1-month horizon. The high-minus-low decile earns on average 0.73% per month ($t = 5.74$). The q-factor alpha is 0.65% ($t = 4.52$), and the q^5 alpha 0.52% ($t = 3.8$). Similarly, the Fama–French six-factor alpha is 0.64% ($t = 4.88$), and the alternative six-factor alpha 0.65% ($t = 4.71$).

The Barillas–Shanken model fails to explain the value-versus-growth anomalies (book-to-market, Bm; earnings-to-price, Ep^{q}12; and sales-to-price, Sp) (Supplementary Appendix). The alphas for the high-minus-low deciles are –0.31, –0.44, and –0.46% per month ($t = -2.39$, -3.6, and -3.11), respectively. In contrast, the Fama–French six-factor alphas are –0.09, –0.03, and –0.18% ($t = -0.82$, -0.26, and -1.38), the q-factor alphas 0.11, –0.07, and –0.09% ($t = 0.71$, -0.44, and -0.48), and the q^5 alphas 0.05, –0.04, and 0.02% ($t = 0.32$, -0.28, and 0.1), respectively.

The culprit is that the UMD loadings in the Barillas–Shanken six-factor model are relatively large, 0.41, 0.19, and 0.19 ($t = 6.84$, 3.08, and 3.83), respectively (untabulated). In contrast, the UMD loadings in the Fama–French six-factor model are small, –0.03, –0.07, and –0.13 ($t = -0.71$, -1.71, and -4.19), respectively. We verify that the correlation between the monthly formed HML^m and UMD is high, –0.65, but that between the annually formed HML and UMD is low, only –0.19. The high HML^m-UMD correlation pushes up the UMD loadings in the presence of HML^m in the Barillas–Shanken model, causing the model to overshoot the average returns to yield large, negative alphas.

The q^5 model largely explains the accruals anomaly. The high-minus-low decile on operating accruals (Oa) has a large q-factor alpha of –0.57% per month ($t = -4.25$). The q^5 model reduces the alpha to –0.2% ($t = -1.3$). A more challenging anomaly for the q-factor model is discretionary accruals (Dac). The high-minus-low Dac decile has a large q-factor alpha of –0.74% ($t = -5.33$), and the q^5 model shrinks the alpha to –0.31%, albeit still significant ($t = -2.16$). For comparison, the Fama–French six-factor alphas for the Oa and Dac deciles are –0.48% ($t = -3.49$) and –0.69% ($t = -5.08$), and the alternative six-factor alphas –0.32% ($t = -2.13$) and –0.59% ($t = -4.12$), respectively.

The q^5 model also improves on the q-factor model in explaining the dWc (change in net noncash working capital) and dFin (change in net financial assets) anomalies. The high-

Table VIII. Explaining the 52 individual anomalies that the *q*-factor model cannot explain (January 1967 to December 2018)

For each high-minus-low decile, we report the average return, $\bar{R}$, the *q*-factor alpha, α_q, the q^5 alpha, α_{q^5}, the Fama–French (2018) six-factor alpha, α_{FF6}, the alpha from the alternative six-factor model with RMW replaced by RMWc, α_{FF6c}, as well as their heteroskedasticity-and-autocorrelation-consistent *t*-statistics, denoted by $t_{\bar{R}}$, t_q, t_{q^5}, t_{FF6}, and t_{FF6c}, respectively. Also, for all the ten deciles formed on a given anomaly variable, we report the mean absolute alphas from the *q*-factor model, $\overline{|\alpha_q|}$, the q^5 model, $\overline{|\alpha_{q^5}|}$, the six-factor model, $\overline{|\alpha_{FF6}|}$, and the alternative six-factor model, $\overline{|\alpha_{FF6c}|}$, as well as the *p*-values from the GRS test on the null hypothesis that all the alphas across a given set of deciles are jointly zero. The *p*-values are denoted by p_q, p_{q^5}, p_{FF6}, and p_{FF6c}, respectively. We also report the loadings on the market, size, investment-to-assets, Roe, and expected growth factors (β_{Mkt}, β_{Me}, $\beta_{I/A}$, β_{Roe}, and β_{Eg}, respectively) in the q^5 model, as well as their heteroskedasticity-and-autocorrelation-adjusted *t*-values (t_{Mkt}, t_{Me}, $t_{I/A}$, t_{Roe}, and t_{Eg}, respectively). Table IV describes the anomaly symbols.

	1 Abr1	2 Abr6	3 Abr12	4 Rs1	5 dEf1	6 $\epsilon^6$6	7 Sm1	8 Ilr1	9 Cm1	10 Sim1	11 Cim1	12 Nop	13 Noa	14 Nsi	15 Cei	16 Ivc	17 Oa	18 dWc	19 dFin	20 Dac
$\bar{R}$	0.73	0.36	0.25	0.36	0.94	0.46	0.50	0.61	0.71	0.78	0.75	0.60	-0.47	-0.67	-0.57	-0.41	-0.29	-0.47	0.27	-0.45
$t_{\bar{R}}$	5.74	3.80	3.23	2.64	4.55	4.03	2.26	3.02	3.65	3.68	3.46	3.30	-3.59	-4.74	-3.42	-3.17	-2.36	-3.70	2.43	-3.47
α_q	0.65	0.34	0.26	0.28	0.56	0.29	0.53	0.65	0.64	0.59	0.66	0.34	-0.50	-0.36	-0.31	-0.26	-0.57	-0.58	0.41	-0.74
α_{q^5}	0.52	0.24	0.18	0.12	0.50	0.08	0.38	0.41	0.60	0.22	0.39	0.18	-0.15	-0.15	-0.03	-0.02	-0.20	-0.23	0.14	-0.31
α_{FF6}	0.64	0.32	0.26	0.44	0.73	0.24	0.52	0.57	0.67	0.61	0.63	0.22	-0.48	-0.31	-0.27	-0.28	-0.48	-0.51	0.46	-0.69
α_{FF6c}	0.65	0.32	0.25	0.41	0.63	0.22	0.49	0.55	0.64	0.57	0.56	0.16	-0.45	-0.25	-0.19	-0.23	-0.32	-0.36	0.34	-0.59
t_q	4.52	3.07	3.08	2.04	2.62	1.99	2.02	2.68	2.72	2.01	2.56	2.50	-3.00	-2.83	-2.49	-1.99	-4.25	-4.38	2.97	-5.33
t_{q^5}	3.80	2.21	1.94	0.90	2.22	0.53	1.34	1.67	2.52	0.73	1.38	1.25	-1.00	-1.12	-0.20	-0.17	-1.30	-1.77	0.97	-2.16
t_{FF6}	4.88	3.70	4.21	3.34	3.88	2.12	2.25	2.66	2.85	2.43	2.78	1.89	-3.44	-2.70	-2.46	-2.35	-3.49	-3.93	3.81	-5.08
t_{FF6c}	4.71	3.48	3.74	3.09	3.20	1.90	1.93	2.36	2.65	2.19	2.49	1.33	-3.07	-2.07	-1.78	-1.84	-2.13	-2.60	2.63	-4.12
$\overline{|\alpha_q|}$	0.12	0.08	0.07	0.07	0.15	0.07	0.12	0.18	0.19	0.14	0.19	0.12	0.12	0.13	0.12	0.07	0.14	0.13	0.08	0.15
$\overline{|\alpha_{q^5}|}$	0.11	0.06	0.05	0.07	0.16	0.06	0.11	0.10	0.16	0.07	0.13	0.11	0.09	0.10	0.07	0.09	0.06	0.10	0.06	0.06

(continued)

Table VIII. Continued

	1 Abr1	2 Abr6	3 Abr12	4 Rs1	5 dEf1	6 ϵ^6	7 Sm1	8 Ilr1	9 Cm1	10 Sim1	11 Cim1	12 Nop	13 Noa	14 Nsi	15 Cei	16 Ivc	17 Oa	18 dWc	19 dFin	20 Dac
$|\alpha_{FF6}|$	0.12	0.06	0.05	0.11	0.18	0.05	0.14	0.17	0.19	0.14	0.19	0.09	0.11	0.12	0.11	0.07	0.12	0.12	0.09	0.14
$|\alpha_{FF6c}|$	0.12	0.06	0.05	0.11	0.17	0.04	0.15	0.17	0.18	0.14	0.18	0.09	0.10	0.12	0.08	0.07	0.07	0.09	0.08	0.12
p_q	0.00	0.00	0.00	0.01	0.00	0.00	0.30	0.07	0.09	0.46	0.00	0.01	0.00	0.00	0.00	0.42	0.00	0.00	0.02	0.00
p_{q^s}	0.00	0.00	0.01	0.05	0.01	0.02	0.58	0.73	0.11	0.99	0.14	0.15	0.01	0.06	0.57	0.26	0.52	0.10	0.60	0.48
p_{FF6}	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.05	0.07	0.41	0.01	0.02	0.00	0.00	0.00	0.22	0.01	0.00	0.01	0.00
p_{FF6c}	0.00	0.00	0.01	0.03	0.01	0.01	0.05	0.07	0.10	0.42	0.02	0.06	0.00	0.00	0.04	0.20	0.29	0.12	0.18	0.00
β_{Mkt}	-0.04	-0.02	0.00	-0.02	0.02	0.00	0.02	-0.14	0.08	0.08	0.02	-0.13	-0.06	0.01	0.17	0.00	0.00	-0.03	0.01	-0.05
β_{Me}	0.06	0.09	0.08	-0.10	-0.05	0.11	-0.18	-0.06	-0.16	0.10	-0.16	-0.31	0.08	0.14	0.24	-0.03	0.25	0.33	-0.08	0.15
$\beta_{I/A}$	-0.19	-0.24	-0.31	-0.51	-0.17	0.02	0.17	0.03	0.24	0.11	0.07	0.99	0.11	-0.56	-0.89	-0.63	0.14	-0.15	-0.36	0.45
β_{Roe}	0.21	0.13	0.12	0.49	0.74	0.14	-0.17	-0.04	-0.06	-0.04	0.05	-0.01	0.17	-0.17	0.00	0.29	0.46	0.32	-0.09	0.38
β_{Eg}	0.19	0.15	0.12	0.24	0.08	0.32	0.24	0.36	0.06	0.56	0.40	0.24	-0.53	-0.31	-0.43	-0.36	-0.56	-0.52	0.40	-0.64
t_{Mkt}	-0.83	-0.59	-0.13	-0.40	0.37	0.07	0.31	-1.99	1.10	1.05	0.35	-3.01	-1.62	0.33	4.94	-0.01	-0.02	-0.77	0.34	-1.62
t_{Me}	0.66	1.86	2.07	-1.99	-0.54	1.78	-1.98	-0.67	-1.94	0.82	-1.71	-3.99	0.80	1.90	3.75	-0.63	4.64	4.03	-1.72	2.83
$t_{I/A}$	-1.89	-3.50	-5.58	-6.51	-1.26	0.27	0.98	0.17	1.47	0.51	0.41	10.17	0.77	-6.97	-12.29	-5.82	1.41	-1.58	-3.03	4.91
t_{Roe}	2.14	1.84	2.60	5.90	6.95	1.42	-0.99	-0.30	-0.35	-0.21	0.38	-0.13	1.68	-2.59	-0.05	3.16	6.43	4.36	-1.05	5.74
t_{Eg}	1.63	1.64	1.70	2.56	0.55	2.93	1.36	2.25	0.35	3.15	2.43	2.32	-5.10	-3.83	-5.04	-3.43	-5.58	-5.45	3.66	-6.02

Table VIII. Continued

	21 Pda	22 dRoe1	23 Ato	24 Ato^{q1}	25 Ato^{q6}	26 Ato^{q12}	27 Opa	28 Ola^{q1}	29 Ola^{q6}	30 Ola^{q12}	31 Cop	32 Cla	33 Cla^{q1}	34 Cla^{q6}	35 Cla^{q12}	36 O^{q1}
$\bar{R}$	−0.56	0.76	0.40	0.67	0.59	0.49	0.47	0.78	0.55	0.51	0.68	0.61	0.52	0.49	0.48	−0.43
$t_{\bar{R}}$	−4.54	5.78	2.32	3.85	3.49	3.02	2.44	3.84	2.85	2.78	3.94	3.65	3.43	3.75	3.88	−1.97
α_q	−0.52	0.36	0.43	0.42	0.41	0.39	0.52	0.43	0.28	0.35	0.75	0.81	0.46	0.41	0.46	−0.38
α_{q^5}	−0.18	0.08	0.10	0.15	0.15	0.14	−0.04	−0.11	−0.23	−0.11	0.11	0.18	−0.04	−0.06	0.03	−0.06
α_{FF6}	−0.48	0.55	0.39	0.44	0.42	0.38	0.54	0.56	0.39	0.42	0.75	0.80	0.50	0.44	0.49	−0.48
α_{FF6c}	−0.45	0.56	0.31	0.40	0.37	0.32	0.44	0.50	0.32	0.35	0.55	0.60	0.43	0.35	0.39	−0.34
t_q	−3.40	2.64	2.82	2.50	2.53	2.51	3.41	2.93	2.11	2.82	5.57	5.78	3.17	3.13	3.83	−2.65
t_{q^5}	−1.22	0.57	0.63	0.88	0.90	0.90	−0.25	−0.84	−2.11	−1.07	0.96	1.57	−0.28	−0.51	0.28	−0.42
t_{FF6}	−3.28	4.49	2.94	2.97	3.08	2.88	3.86	3.94	3.24	3.84	6.44	6.71	3.79	3.96	4.79	−3.26
t_{FF6c}	−2.99	4.36	2.23	2.57	2.51	2.30	2.87	3.05	2.23	2.69	4.75	5.16	3.17	3.01	3.76	−2.36
$\overline{\lvert\alpha_q\rvert}$	0.18	0.09	0.09	0.11	0.07	0.07	0.14	0.13	0.09	0.09	0.18	0.15	0.20	0.12	0.13	0.08
$\overline{\lvert\alpha_{q^5}\rvert}$	0.09	0.06	0.12	0.12	0.12	0.12	0.07	0.07	0.07	0.05	0.07	0.07	0.07	0.05	0.04	0.08
$\overline{\lvert\alpha_{FF6}\rvert}$	0.16	0.10	0.10	0.11	0.08	0.07	0.14	0.18	0.13	0.12	0.18	0.15	0.20	0.12	0.13	0.10
$\overline{\lvert\alpha_{FF6c}\rvert}$	0.13	0.09	0.11	0.13	0.09	0.09	0.14	0.18	0.13	0.12	0.15	0.14	0.18	0.11	0.13	0.10
p_q	0.00	0.03	0.00	0.01	0.04	0.03	0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.01	0.00	0.01
p_{q^5}	0.17	0.45	0.00	0.01	0.01	0.01	0.08	0.52	0.14	0.38	0.25	0.40	0.49	0.93	0.51	0.25
p_{FF6}	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
p_{FF6c}	0.01	0.01	0.01	0.00	0.02	0.03	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.08	0.00	0.05
β_{Mkt}	−0.03	0.06	0.24	0.15	0.13	0.12	−0.16	−0.03	−0.03	−0.06	−0.13	−0.11	−0.01	0.03	0.00	0.09
β_{Me}	0.05	−0.01	0.29	0.43	0.38	0.33	−0.40	−0.27	−0.32	−0.32	−0.53	−0.55	−0.27	−0.27	−0.27	0.77
$\beta_{I/A}$	−0.09	0.10	−1.08	−0.62	−0.73	−0.80	−0.58	−0.43	−0.48	−0.57	−0.32	−0.56	−0.29	−0.28	−0.32	0.28
β_{Roe}	0.14	0.48	0.16	0.43	0.41	0.36	0.42	0.81	0.73	0.65	0.20	0.12	0.22	0.22	0.18	−0.62
β_{Eg}	−0.50	0.41	0.50	0.40	0.38	0.36	0.82	0.84	0.79	0.72	0.97	0.93	0.77	0.73	0.66	−0.49
t_{Mkt}	−0.82	1.56	5.53	2.66	2.50	2.30	−4.36	−0.77	−1.02	−2.37	−3.75	−3.07	−0.26	1.21	−0.20	2.39
t_{Me}	0.66	−0.08	5.05	5.54	5.56	5.81	−4.83	−3.69	−5.72	−5.76	−8.09	−9.19	−4.57	−5.84	−6.20	14.54
$t_{I/A}$	−0.70	1.22	−9.62	−6.62	−7.92	−8.95	−7.09	−4.75	−6.48	−7.94	−4.42	−7.01	−3.19	−3.48	−4.52	2.78
t_{Roe}	1.56	5.28	2.17	4.30	5.15	4.71	6.09	10.71	11.84	9.43	3.59	1.94	3.13	4.28	3.77	−8.65
t_{Eg}	−4.78	3.61	4.59	3.49	3.49	3.35	8.07	9.14	10.39	8.73	11.84	12.27	7.00	10.18	10.81	−5.75

Table VIII. Continued

	37 Tbi^{q}12	38 Rdm	39 Rdm^{q}1	40 Rdm^{q}6	41 Rdm^{q}12	42 Rds^{q}6	43 Rds^{q}12	44 Rer	45 Eprd	46 R_a^1	47 $R_a^{[2,5]}$	48 $R_a^{[6,10]}$	49 $R_a^{[11,15]}$	50 $R_a^{[16,20]}$	51 Isff1	52 Isq1
$\bar{R}$	0.21	0.73	1.09	0.80	0.83	0.50	0.51	0.39	−0.59	0.63	0.71	0.81	0.63	0.57	0.30	0.22
$t_{\bar{R}}$	2.02	2.96	3.04	2.31	2.62	2.00	2.01	2.85	−3.38	3.31	4.31	5.09	4.65	3.53	3.41	2.59
α_q	0.32	0.81	1.41	1.02	0.92	0.90	0.93	0.40	−0.58	0.53	0.83	1.08	0.61	0.65	0.31	0.28
α_{q^5}	0.36	0.27	1.05	0.58	0.43	0.64	0.65	0.23	−0.48	0.43	0.84	0.91	0.56	0.63	0.23	0.18
α_{FF6}	0.22	0.68	1.36	1.01	0.88	0.88	0.93	0.32	−0.79	0.42	0.76	1.08	0.66	0.62	0.29	0.23
α_{FF6c}	0.15	0.79	1.37	1.06	0.96	0.98	1.01	0.30	−0.84	0.34	0.69	1.06	0.67	0.65	0.28	0.21
t_q	2.94	3.64	3.33	3.25	3.55	3.27	3.36	2.51	−3.32	2.57	4.28	5.13	3.68	3.48	3.05	2.84
t_{q^5}	3.01	1.24	2.37	1.79	1.60	2.31	2.35	1.46	−2.83	1.94	4.11	4.62	3.27	3.06	2.07	1.71
t_{FF6}	1.98	3.24	3.90	3.48	3.56	3.91	4.10	2.12	−5.04	2.39	4.00	5.61	4.28	3.62	3.17	2.45
t_{FF6c}	1.34	3.64	3.93	3.71	3.98	4.44	4.54	1.99	−5.23	1.84	3.49	5.14	4.00	3.49	2.93	2.16
$|\alpha_q|$	0.10	0.28	0.53	0.47	0.46	0.30	0.30	0.13	0.15	0.14	0.17	0.24	0.17	0.16	0.09	0.11
$|\alpha_{q^5}|$	0.08	0.12	0.36	0.27	0.24	0.23	0.21	0.12	0.16	0.12	0.17	0.20	0.17	0.16	0.08	0.09
$|\alpha_{FF6}|$	0.09	0.24	0.48	0.41	0.40	0.28	0.28	0.11	0.19	0.12	0.15	0.24	0.18	0.16	0.08	0.08
$|\alpha_{FF6c}|$	0.09	0.24	0.46	0.40	0.39	0.26	0.26	0.11	0.21	0.11	0.14	0.24	0.19	0.18	0.08	0.07
p_q	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.11	0.00	0.00	0.00	0.00	0.00	0.00
p_{q^5}	0.01	0.25	0.00	0.02	0.03	0.00	0.00	0.05	0.01	0.53	0.00	0.00	0.00	0.02	0.02	0.06
p_{FF6}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.29	0.00	0.00	0.00	0.01	0.01	0.01
p_{FF6c}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.41	0.00	0.00	0.00	0.00	0.02	0.08
β_{Mkt}	−0.08	0.23	0.08	0.00	0.01	−0.08	−0.11	0.09	0.10	0.23	0.06	0.00	0.00	−0.06	−0.01	−0.01
β_{Me}	−0.17	0.67	0.21	0.57	0.67	0.20	0.18	−0.11	0.34	−0.14	−0.17	0.04	−0.06	−0.08	0.14	0.20
$\beta_{I/A}$	−0.12	−0.10	0.47	0.51	0.61	−1.00	−1.01	−0.16	0.47	−0.21	−0.30	−0.42	−0.02	−0.06	−0.07	−0.10
β_{Roe}	0.04	−0.87	−1.15	−1.06	−0.90	−0.41	−0.41	−0.02	−0.57	0.14	0.04	−0.30	0.07	−0.01	−0.08	−0.16
β_{Eg}	−0.05	0.84	0.55	0.67	0.75	0.40	0.42	0.24	−0.14	0.15	−0.02	0.25	0.08	0.02	0.12	0.15
t_{Mkt}	−2.16	3.93	0.71	0.05	0.15	−1.01	−1.30	1.69	1.73	4.34	1.01	−0.06	−0.07	−1.28	−0.18	−0.28
t_{Me}	−3.34	7.55	1.07	4.24	5.76	1.33	1.15	−1.19	4.45	−1.23	−1.67	0.47	−0.66	−1.53	3.75	2.80
$t_{I/A}$	−1.89	−0.69	1.65	2.58	3.68	−6.09	−6.55	−1.36	4.01	−1.46	−2.65	−2.62	−0.18	−0.48	−0.96	−1.42
t_{Roe}	0.58	−5.83	−4.15	−6.22	−6.26	−2.27	−2.42	−0.21	−5.13	0.99	0.32	−2.34	0.63	−0.13	−1.40	−2.76
t_{Eg}	−0.67	5.37	2.45	3.50	4.61	2.45	2.67	1.98	−1.25	1.02	−0.19	1.88	0.66	0.21	1.81	2.03

minus-low dWc and dFin deciles have significant q-factor alphas of –0.58% per month ($t = -4.38$) and 0.41% ($t = 2.97$) but insignificant q^5 alphas of –0.23% ($t = -1.77$) and 0.14% ($t = 0.97$), respectively. For comparison, the Fama–French six-factor alphas are –0.51% ($t = -3.93$) and 0.46% ($t = 3.81$), and the alternative six-factor alphas –0.36% ($t = -2.6$) and 0.34% ($t = 2.63$), respectively.

The high-minus-low Oa and Dac deciles have large expected growth factor (R_{Eg}) loadings of –0.56 ($t = -5.58$) and –0.64 ($t = -6.02$), respectively. As such, high operating and discretionary accruals indicate low expected growth. Intuitively, given the level of earnings, high accruals mean low cash flows available for financing investments, giving rise to low expected growth. Similarly, the high-minus-low dWc decile has a large R_{Eg} loading of –0.52 ($t = -5.45$). Intuitively, increases in net noncash working capital signal high past growth but low expected growth. Finally, the high-minus-low dFin decile has a large R_{Eg} loading of 0.4 ($t = 3.66$). Intuitively, increases in net financial assets provide more internal funds available for investments, stimulating expected growth going forward.

The q^5 model largely explains the R&D-to-market (Rdm) anomaly. The annually sorted high-minus-low decile has a q-alpha of 0.81% per month ($t = 3.64$). The q^5 model reduces the alpha to 0.27% ($t = 1.24$) via a large R_{Eg} loading of 0.84 ($t = 5.37$). Similarly, in monthly sorts, at the 1-, 6-, and 12-month horizons, the high-minus-low Rdm^q deciles have q-alphas of 1.41, 1.02, and 0.92% ($t = 3.33, 3.25$, and 3.55) but smaller q^5 alphas of 1.05, 0.58, and 0.43% ($t = 2.37, 1.79$, and 1.6), respectively. The corresponding R_{Eg} loadings are 0.55, 0.67, and 0.75 ($t = 2.45, 3.5$, and 4.61), respectively. Intuitively, R&D expenses depress current earnings due to current accounting standards but raise intangible capital that induces future growth opportunities. While the q-factor model misses this economic mechanism, the q^5 model with the expected growth factor accommodates it. For comparison, the high-minus-low Rdm decile has a Fama–French six-factor alpha of 0.68% per month ($t = 3.24$) and an alternative six-factor alpha of 0.79% ($t = 3.64$). The high-minus-low Rdm^q deciles have six-factor alphas of 1.36, 1.01, and 0.88% ($t = 3.9, 3.48$, and 3.56), as well as alternative six-factor alphas of 1.37, 1.06, and 0.96% ($t = 3.93, 3.71$, and 3.98), respectively.

5. Conclusion

In the investment theory, firms with high expected investment growth should earn higher expected returns than firms with low expected investment growth, holding current investment and expected profitability constant. Motivated by this economic insight, we form cross-sectional growth forecasts and construct an expected growth factor, which yields an average premium of 0.84% per month ($t = 10.27$) in the 1967–2018 monthly sample. We augment the q-factor model with the expected growth factor to form the q^5 model. In a large set of testing deciles based on 150 anomalies, the q^5 model shows strong explanatory power and substantially outperforms the Fama–French six-factor model.

Supplementary Material

Supplementary data are available at *Review of Finance* online.

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